

## REINFORCED CONCRETE DESIGN CIVIL 4TH SEM. LEARNING OUTCOMES

After undergoing the subject, students will be able to:

- Explain methods of RCC design i.e.
  - Working stress methods
  - Limit state methods
- Design singly, doubly reinforced rectangular and T&L beams as per IS Code
- Design one way and two way slab
- Design axially loaded column and their isolated footing

### UNIT-1 GENERAL FEATURES OF REINFORCED CONCRETE:

#### 1.1 Introduction:

A structure refers to a system of connected parts used to support forces (loads). Buildings, bridges and towers are examples for structures in civil engineering. In buildings, structure consists of walls floors, roofs and foundation. In bridges, the structure consists of deck, supporting systems and foundations. In towers the structure consists of vertical, horizontal and diagonal members along with foundation.

A structure can be broadly classified as (i) sub structure and (ii) super structure. The portion of building below ground level is known as sub-structure and portion above the ground is called as super structure. Foundation is sub structure and plinth, walls, columns, floor slabs with or without beams, stairs, roof slabs with or without beams etc are super structure.

Many naturally occurring substances, such as clay, sand, wood, rocks natural fibers are used to construct buildings. Apart from this many manmade products are in use for building construction. Bricks, tiles, cement concrete, concrete blocks, plastic, steel & glass etc are manmade building materials.

Cement concrete is a composite building material made from combination of aggregates (coarse and fine) and a binder such as cement. The most common form of concrete consists of mineral aggregate (gravel & sand), Portland cement and water. After mixing, the cement hydrates and eventually hardens into a stone like material. Recently a large number of additives known as concrete additives are also added to enhance the quality of concrete. Plasticizers, super plasticizers, accelerators, retarders, pozzolonic materials, air entraining agents, fibers, polymers and silica fumes are the additives used in concrete. Hardened concrete has high compressive strength and low tensile strength. Concrete is generally strengthened using steel bars or rods known as rebars in tension zone. Such elements are “reinforced concrete” concrete can be moulded to any complex shape using suitable form work and it has high durability, better appearance, fire resistance and economical. For a strong, ductile and durable construction the reinforcement shall have high strength, high tensile strain and good bond to concrete and thermal compatibility. Building components like slab walls, beams, columns foundation & frames are constructed with reinforced concrete. Reinforced concrete can be in-situ concreted or precast concrete.

For understanding behavior of reinforced concrete, we shall consider a plain concrete beam subjected to external load as shown in Fig. 1.1. Tensile strength of concrete is approximately

one-tenth of its compressive strength.

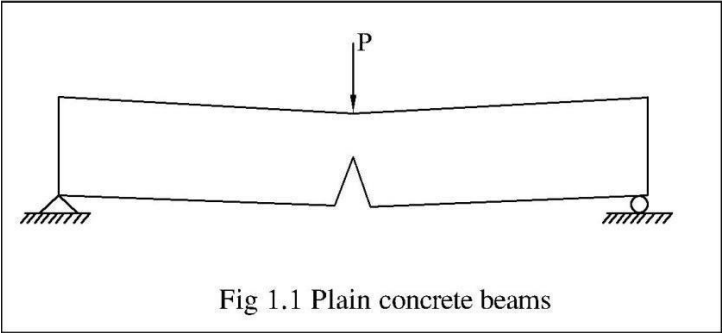


Fig 1.1 Plain concrete beams

Hence use of plain concrete as a structural material is limited to situations where significant tensile stresses and strains do not develop as in solid or hollow concrete blocks, pedestal and in mass concrete dams. The steel bars are used in tension zone of the element to resist tension as shown in Fig 1.2. The tension caused by bending moment is chiefly resisted by the steel reinforcements, while concrete resists the compression. Such joint action is possible if relative slip between concrete and steel is prevented. This phenomenon is called “bond”. This can be achieved by using deformed bars which have high bond strength at the steel-concrete interface. Rebar imparts “ductility” to the structural element, i.e. RC elements have large deflection before they fail due to yielding of steel, thus they give ample warning before their collapse.

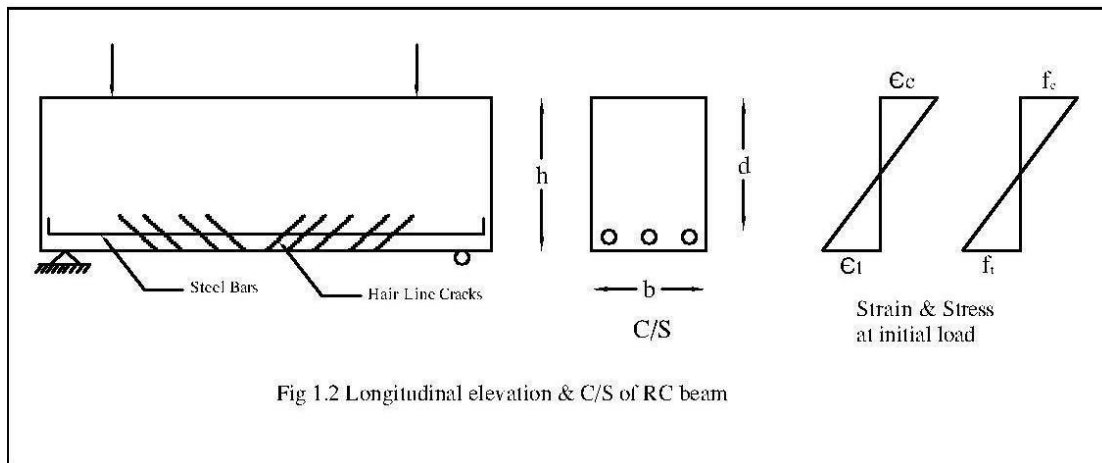


Fig 1.2 Longitudinal elevation & C/S of RC beam

## 1.2 Design Loads

For the analysis and design of structure, the forces are considered as the “Loads” on the structure. In a structure all components which are stationary, like wall, slab etc., exert forces due to gravity, which are called as “Dead Loads”. Moving bodies like furniture, humans etc exert forces due to gravity which are called as “Live Loads”. Dead loads and live loads are gravity forces which act vertically down ward. Wind load is basically a horizontal force due to wind pressure exerted on the structure. Earthquake load is primarily a horizontal pressure exerted due to movement of the soil on the foundation of a structure. Vertical earthquake force is about 5% to 10% of horizontal earthquake force. Fig. 1.3 illustrates the loads that are considered in analysis and design.

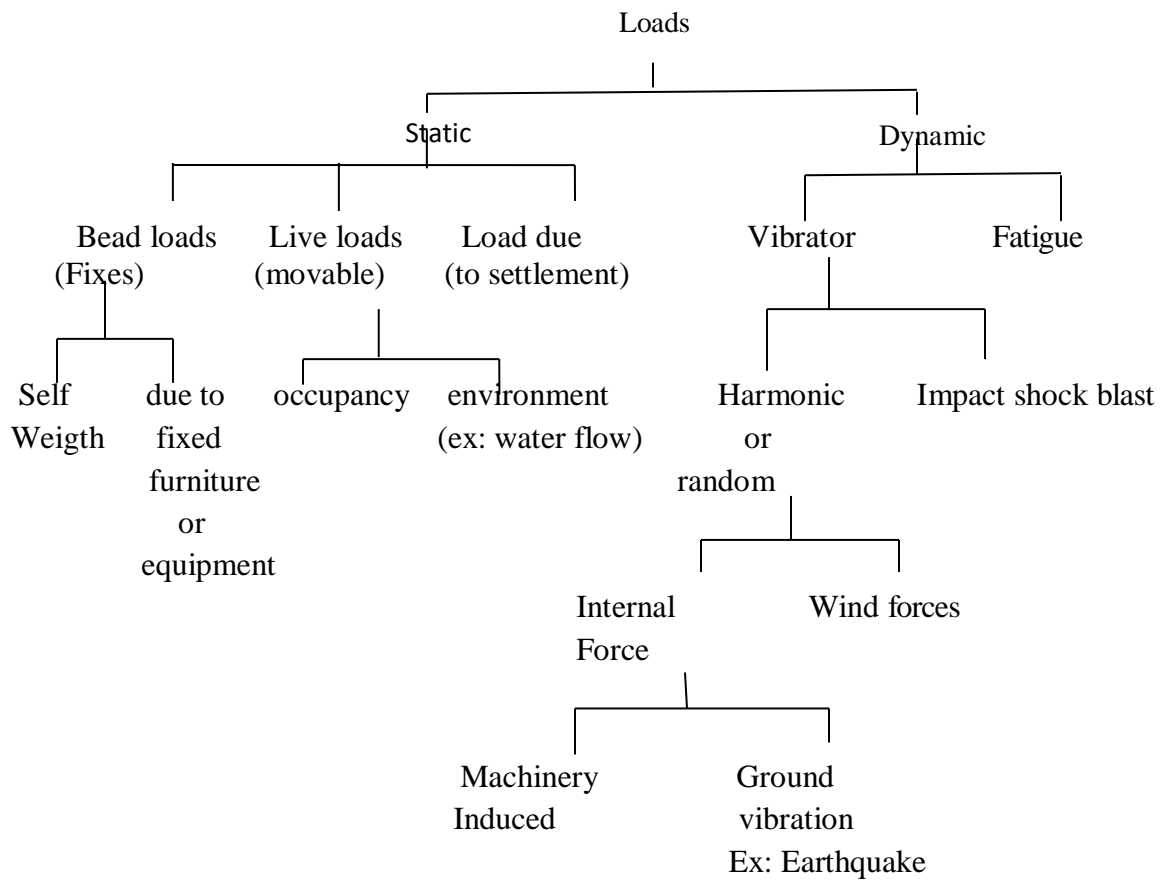


Fig 1.3 Types of loads on Structure

IS875 -1987 part 1 gives unit weight of different materials, Part – 2 of this code describes live load on floors and roof. Wind load to be considered is given in part 3 of the code. Details of earthquake load to be considered is described in 1893 – 2002 code and combination of loads is given in part 5 of IS875 – 1987.

### 1.3. Materials for Reinforced Concrete

#### Concrete

Concrete is a composite material consists essentially of

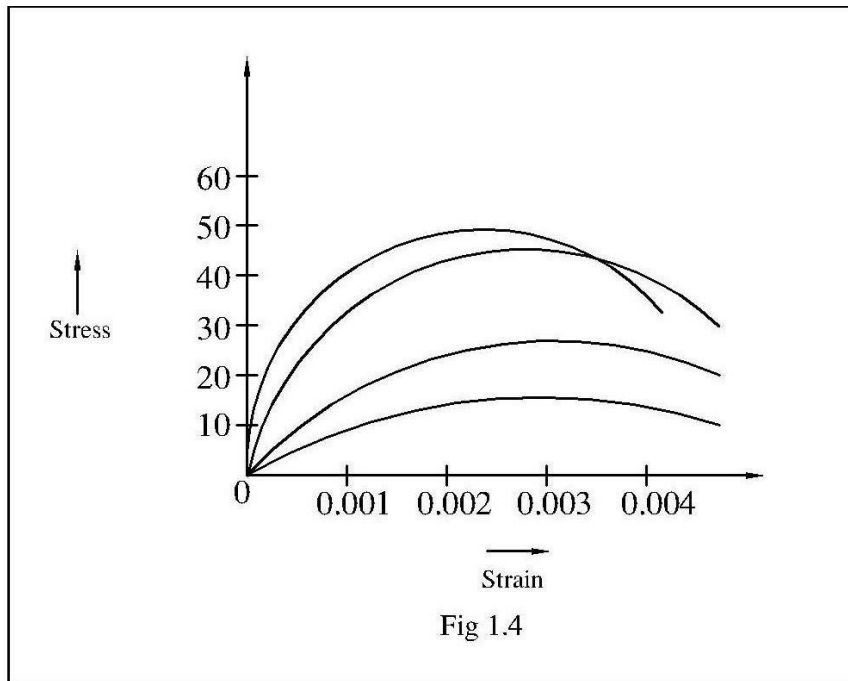
- a) A binding medium cement and water called cement paste
- b) Particles of a relatively inert filler called aggregate

The selection of the relative proportions of cement, water and aggregate is called “mix design” Basic requirement of a good concrete are workability, strength, durability and economy. Depending upon the intended use the cement may be OPC (33,43 & 53 Grade), Rapid hardening cements Portland slag, Portland pozzolona etc. High cement content give rise to increased shrinkage, creep and cracking. Minimum cement content is  $300\text{Kg/m}^3$  and maximum being  $450\text{Kg/m}^3$  as per Indian code. Mineral additives like fly ash , silica fume, rice husk ash, metakoline and ground granulated blast furnace slag may be used to reduce micro cracks . The aggregate used is primarily for the purpose of providing bulk to the concrete and constitutes 60 to 80 percent of finished product. Fine aggregates are used to increase the workability and uniformly of concrete mixture. Water used for mixing and curing shall be clean and free from oil, acids, alkalis, salts, sugar etc. The diverse requirements of mixability, stability, transportability place ability, mobility, compatibility of fresh concrete are collectively referred to as workability.

Compressive strength of concrete on 28<sup>th</sup> day after casting is considered as one of the measure of quality. At least 4 specimens of cubes should be tested for acceptance criteria.

#### Grade of concrete

Based on the compressive strength of concrete, they are designated with letter H followed by an integer number represented characteristic strength of concrete, measured using 150mm size cube. Characteristic strength is defined us the strength of material below which not more than 5% of test results are expected to full. The concrete grade M10, M15 and M20 are termed as ordinary concrete and those of M25 to M55 are termed as standard concrete and the concrete of grade 60 and above are termed as high strength concrete. The selection of minimum grade of concrete is dictated by durability considerations which are based on kind of environment to which the structure is exposed, though the minimum grade of concrete for reinforced concrete is specified as M20 under mild exposure conditions, it is advisable to adopt a higher grade. For moderate, severe, very severe and extreme exposure conditions, M25, M30, M35 & M40 grades respectively are recommended. Typical stress-strain curves of concrete is shown in Fig.1.4



### Reinforcing steel

Steel bars are often used in concrete to take care of tensile stresses. Often they are called as rebars, steel bar induces ductility to composite material i.e reinforced concrete steel is stronger than concrete in compression also. Plain mild steel bars or deformed bars are generally used. Due to poor bond strength plain bars are not used. High strength deformed bars generally cold twisted (CTD) are used in reinforced concrete. During beginning of 21st century, Thermo- mechanical treatment (TMT) bars which has ribs on surface are used in reinforced concrete. Yield strength of steel bars are denoted as characteristic strength. Yield strength of mild steel is 250MPa, yield strength of CTD &TMT bars available in market has 415 MPa or 500 MPa or 550MPa. TMT bars have better elongation than CTD bars. Stress-strain curve of CTD bars or TMT bars do not have definite yield point, hence 0.2% proof stress is used as yield strength. Fig

1.5 shows stress strain curve of different steel grades. Steel grades are indicated by Fe followed by yield strength. In the drawings of RCC,  $\square$  denotes MS bar and # denotes CTD or TMT bars

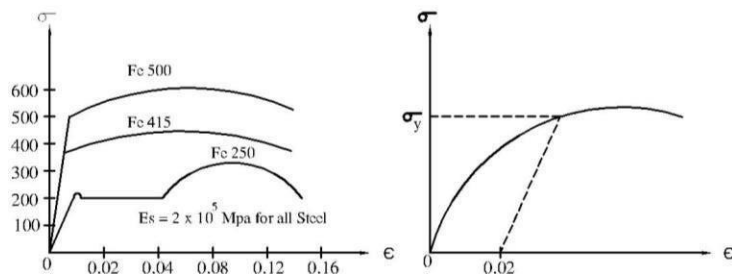


Fig 1-5 stress –strain curve

#### 1.4 Design codes and Hand books

A code is a set of technical specifications intended to control the design and construction. The code can be legally adopted to see that sound structures are designed and constructed code specifies acceptable methods of design and construction to produce safe and sound structures.

National building code have been formulated in different countries to lay down guidelines for the design and construction of structures. International building code has been published by international code council located in USA. National building code (NBC – 2005) published in India describes the specification and design procedure for buildings.

For designing reinforced concrete following codes of different countries are available

India – IS456 – 2000 – Plain and reinforced concrete code practice.

USA - ACI 318-2011 – Building code requirements for Structural concrete (American concrete institute)

UK - BS8110–part1 – structural use of concrete –code of practice for design and construction. (British standard Institute)

Europe – EN 1992(Euro code 2) - Design of concrete structures

Canada – CAN/CSA – A23.3-04 - Design of concrete structures (Reaffirmed in 2010), Australia – As 3600 -2001 – concrete structures.

Germany – Din 1045 – Design of concrete structures

Russia – SNIP

China - GB 50010 -2002 code for design of concrete structures to help the designers, each country has produced “handbook”. In India following hand books called special publication are available.

SP – 16-1980- Design Aid for Reinforced concrete to IS456-

1978 SP – 23-1982- Hand book on concrete mixes

Sp – 24 -1983 – Explanatory hand book on IS456 – 1978

SP – 34-1987 - Hand book on concrete reinforcement and detailing.

### 1.5 Design Philosophies

Structural design is process of determining the configuration (form and proportion) of a structure subject to a load carrying performance requirement. Form of a structure describes the shape and relative arrangements of its components. The determination of an efficient form is basically a trial and error procedure.

In the beginning of 20<sup>th</sup> century (1900 to 1960) to late 50's of this century, members were proportioned so that stresses in concrete and steel resulting from service load were within the allowable stresses. Allowable stresses were specified by codes. This method of design is called “working stress method” (WSM). This method of design resulted in conservative sections and was not economical. This design principle satisfies the relation — .

Where R is resistance of structural element, RS is factor of safety and L is applied external load.

In 1950's ultimate load method or load factor method was developed. In this method, using non linear stress – strain curve of concrete and steel, the resistance of the element is computed. The safety measure in the design is introduced by an appropriate choice of the load factor (ultimate load/working load). Different load factors are assigned for different loads. Following equations are used for finding ultimate load as per IS456 – 1964

$$U = 1.5 DL + 2.2 LL$$

$$U = 1.5 DL + 2.2LL \text{ to } 5WL \text{ or } 1.5 DL + 0.5LL + 2.2 WL$$

Here DL = Dead load, LL = Live load WL= wind load or earthquake load. The design principle should satisfy  $R \geq LF$  etc or  $R \geq U$ , Where, R= Resistance, LF= Load factor, L= load. Ultimate load method generally results in more slender section, but leads to larger deformation. Due to the disadvantage of larger deflection, this method was discontinued. To overcome the disadvantages of working stress method and ultimate method, a probabilistic design concept called as “Limit state method, was developed during 1970's. IS456 -1978 recommended this method and is continued in 2000 version also. This method safe guards the risk of both collapse and unserviceability. Limit state method uses multiple safety factor format, which attempt to provide adequate safety at ultimate loads or well as a denote serviceability at



service loads by considering all limit states, The acceptable limit for safety and serviceability requirements before failure or collapse is termed as “ Limit state” Two principal limit states are considered i.e 1. Limit state of collapse 2. Limit state of serviceability. The limit state of collapse include one or more of i) flexure, II) shear, III) torsion and IV) compression the limit state of collapse is expressed as  $\mu R > \gamma X_i L_i$  Where,  $\mu$  and  $\gamma$  are partial safety factors, Here  $\mu < 1$  &

$\gamma > 1$ . The most important limit state considered in design are of deflection, other limit state of serviceability are crack and vibration. For deflection  $\gamma_{max} \leq$  where  $\gamma_{max}$  is maximum

deflection,  $l = \text{span} / 4$  is an integer numbers. For over all deflection is 250 and for short term deflection = 350.

### 1.6 Partial safety factor

To account for the different conditions like for material strength, load etc. Different partial factors are used for material and load.  $\gamma_m$  indicate safety factor for material & for load

Design strength = \_\_\_\_\_

Design load =  $\gamma_f$  x characteristic load

As per clause 36.4.2 page 68 of IS 456,  $\gamma_m = 1.5$  for concrete and  $\gamma_m = 1.15$  for steel. Similarly clause 36.4.1 page 68 of code gives  $\gamma_f$  in table 18 for different values for different load combinations and different limit states.

IS 456 – 2000 Recommendations

(i) Partial safety factors for materials to be multiplied with characteristic strength is given below.

Values of partial safety factor  $\gamma_m$

Material	Limit state		
	Collapse	Deflection	Cracking
Concrete	1.5	1.0	1.3
Steel	1.15	1.0	1.0

Design strength = \_\_\_\_\_

- (ii) Partial safety factors for loads to be multiplied with characteristic load is given below.

Value of partial safety factors  $f$

Load combination	Ultimate limit state	Serviceability limit state
1) Dead load & live load	$1.5(DL+LL)$	$DL+LL$
2) Dead seismic/wind load	$0.9DL+1.5(E2/WL)$	$DL +$
a) Dead load contributes to Stability		$EQ/WL$
b) Dead load assists overturning	$1.5(DL+E2/WL)$	$DL+EQ/wL$
3) Dead, live load and Seismic/wind load	$1.2(DL+LL+EQ/WL)$	$Dl+0.LL+0.8EQ/WL$

DL-Dead load, LL- Live load WL- Wind load EQ- Earthquake load

- (iii) The code has suggested effective span to effective depth ratios as given below

Basic effective span to effective depth ratio ( $l/l$ ) basic

Type of beam one /slab	Span $\leq$ 10m	Span $>$ 10m
1)Cantilever	7	Deflection should be Be calculated
2) Simply supported	20	$(20 \times 10)/\text{span}$
3)continuous beam	26	$(26 \times 10)/\text{span}$

The above values are to be modified for (i) the type and amount of tension steel (Fig 4 page 38 of T5456-2000)

- (ii) The amount of compression steel (Fig 5 page 39 of I5456-2000)

- (iv) The type of beam ie flanged beams etc (Fig 6 page 39 of I5456 – 2000).

For slabs spanning in two directions, the l/d ratio is given below.

For slabs spanning in two directions, the l/d ratio is given below

Type of slab	l/d for grade of steel	
	Fe250	Fe415
1) Simply supported	35	28
2) Continuous	40	32

### 1.7 Characteristic strength and loads

Limit state method is based on statistical concepts. Strength of materials and loads are highly variable in a range of values. The test in laboratory on compressive strength of concrete has indicated coefficient of variation of  $\pm 10\%$ . Hence in reinforced concrete construction, It is not practicable to specify a precise cube strength. Hence in limit state design uses the concept of “characteristic strength”  $f_{ck}$  indicates characteristics strength of concrete & by characteristic strength of steel. In general  $f_k$  indicates the characteristic strength of material.

$$f_k = f_m - 1.646 \sigma \quad (2.6) \text{ here } f_m = \text{mean strength.}$$

Similarly “characteristic load” is that value of load which has an accepted probability of not being exceeded during the life span of structure. In practice the load specified by IS875 – 1987 is considered as characteristic load. Equation for characteristic load is

$$L_k = L_m + 1.64$$

## UNIT-2

### PRINCIPLES OF LIMIT STATE DESIGN AND ULTIMATE STRENGTH OF R.C. SECTION:

#### 2.1 Introduction:

A beam experiences flexural stresses and shear stresses. It deforms and cracks are developed. RC beam should have perfect bond between concrete and steel for composite action. It is primarily designed as flexural member and then checked for other parameters like shear, bond, deflection etc. In reinforced concrete beams, in addition to the effects of shrinkage, creep and loading history, cracks developed in tension zone effects its behavior. Elastic design method (WSM) do not give a clear indication of their potential strengths. Several investigators have published behavior of RC members at ultimate load. Ultimate strength design for beams was introduced into both the American and British code in 1950's. The Indian code IS456 introduced the ultimate state method of design in 1964. Considering both probability concept and ultimate load called as "Limit state method of design" was introduced in Indian code from 1978.

#### 2.2 Behavior of Reinforced concrete beam

To understand the behavior of beam under transverse loading, a simply supported beam subjected to two point loading as shown in Fig. 2.1 is considered. This beam is of rectangular cross-section and reinforced at bottom.

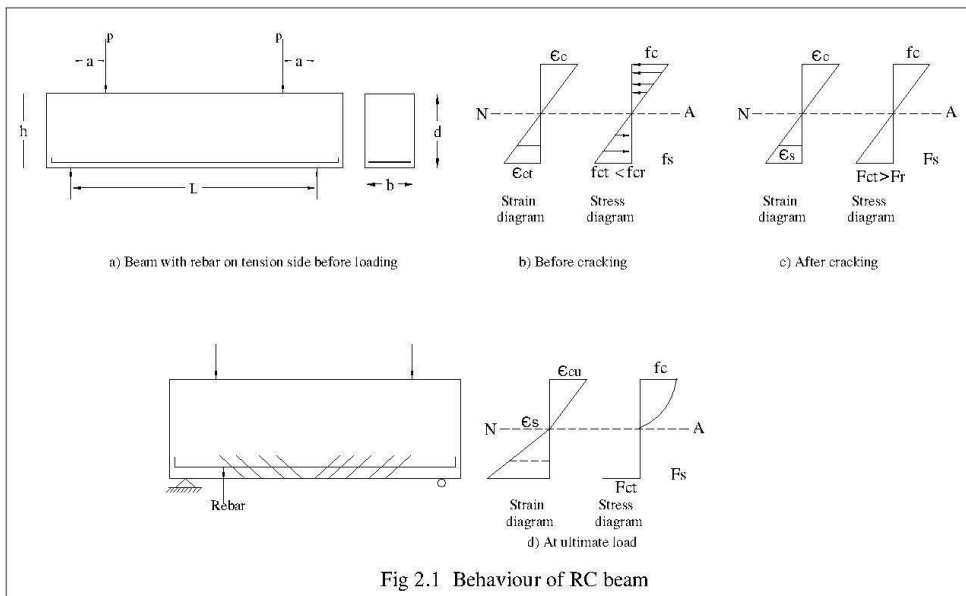


Fig 2.1 Behaviour of RC beam

When the load is gradually increased from zero to the ultimate load value, several stages of behavior can be observed. At low loads where maximum tensile stress is less than modulus of rupture of concrete, the entire concrete is effective in resisting both compressive stress and tensile stress. At this stage, due to bonding tensile stress is also induced in steel bars.

With increase in load, the tensile strength of concrete exceeds the modulus of rupture of concrete and concrete cracks. Cracks propagate quickly upward with increase in loading up to neutral axis. Strain and stress distribution across the depth is shown in Fig 4.1c. Width of crack is small. Tensile stresses developed are absorbed by steel bars. Stress and strain are proportional till  $f_c < f_{cr}$ . Further increase in load, increases strain and stress in the section and are no longer proportional. The distribution of stress – strain curve of concrete. Fig 4.1d shows the stress distribution at ultimate load.

Failure of beam depends on the amount of steel present in tension side. When moderate amount of steel is present, stress in steel reaches its yielding value and stretches a large amount with tension crack in concrete widens. Cracks in concrete propagate upward with increases in deflection of beam. This induces crushing of concrete in compression zone and called as “secondary compression failure”. This failure is gradual and is preceded by visible signs of distress. Such sections are called “under reinforced” sections.

When the amount of steel bar is large or very high strength steel is used, compressive stress in concrete reaches its ultimate value before steel yields. Concrete fails by crushing and failure is sudden. This failure is almost explosive and occur without warning. Such reactions are called “over reinforced section”

If the amount of steel bar is such that compressive stress in concrete and tensile stress in steel reaches their ultimate value simultaneously, then such reactions are called “Balanced Section”.

The beams are generally reinforced in the tension zone. Such beams are termed as “singly reinforced” section. Some times rebars are also provided in compression zone in addition to tension rebars to enhance the resistance capacity, then such sections are called “Doubly reinforced section.”

### 2.3 Assumptions

Following assumptions are made in analysis of members under flexure in limit state method

1. Plane sections normal to axis remain plane after bending. This implies that strain is proportional to the distance from neutral axis.
2. Maximum strain in concrete of compression zone at failure is 0.0035 in bending.

3. Tensile strength of concrete is ignored.
4. The stress-strain curve for the concrete in compression may be assumed to be rectangle, trapezium, parabola or any other shape which results in prediction of strength in substantial agreement with test results. Design curve given in IS456-2000 is shown in Fig. 2.2

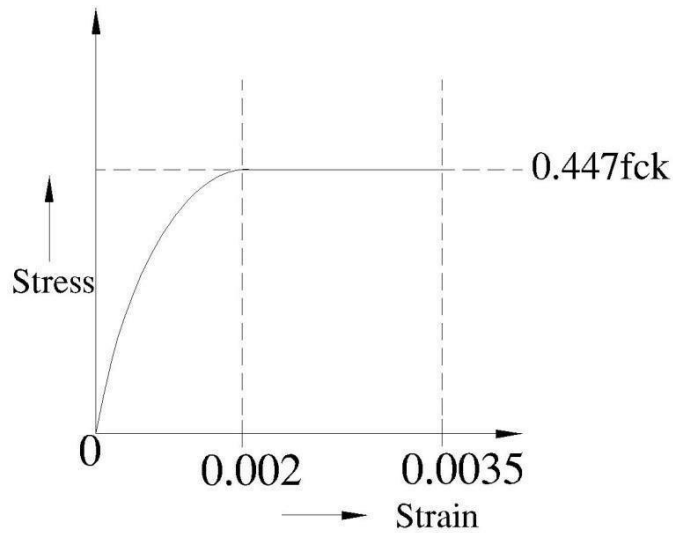


Fig 2.2 Stress-Strain Curve for Concrete

5. Stress – strain curve for steel bar with definite yield point and for cold worked deformed bars is shown in Fig 2.3 and Fig 2.4 respectively.

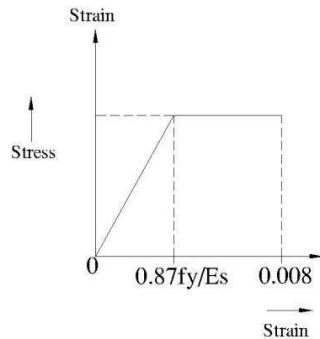


Fig 2.3 stress-strain curve for steel bar with defective yield point

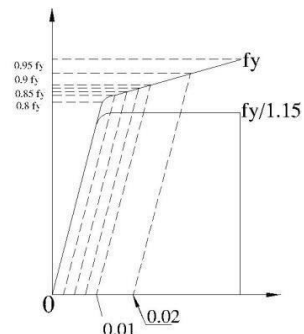


Fig 2.4 stress-strain curve for cold worked deformed bars

6. To ensure ductility, maximum strain in tension reinforcement shall not be less than

\_\_\_\_\_

7. Perfect bond between concrete and steel exists.

#### 2.4. Analysis of singly reinforced rectangular sections

Consider a rectangular section of dimension  $b \times h$  reinforced with  $A_{st}$  amount of steel on tension side with effective cover  $C_e$  from tension extreme fiber to C.G of steel. Then effective depth  $d=h-c_e$ , measured from extreme compression fiber to C.G of steel strain and stress distribution across the section is shown in Fig.2.4. The stress distribution is called stress block.

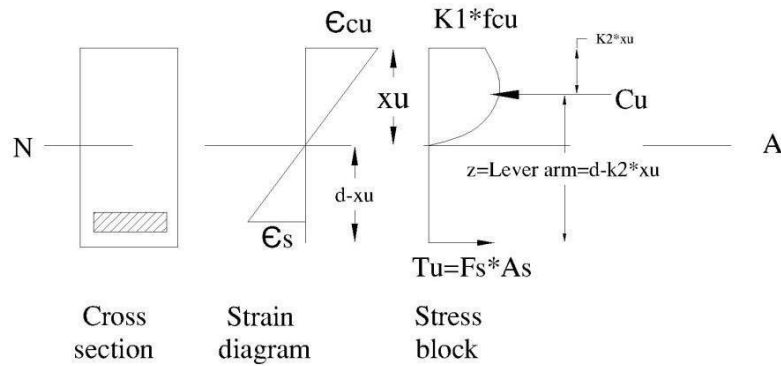


Fig 2.5 Stress Block

From similar triangle properly applied to strain diagram

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\_\_\_\_\_

For the known value of  $x_u$  &  $\epsilon_{cu}$  the strain in steel is used to get the value of stress in steel from stress-strain diagram. Equation 4.4-1 can be used to get the value of neutral axis depth as

\_\_\_\_\_

( — )

( — )

□ —

Here — is called neutral axis factor

For equilibrium  $C_u = T_u$ .

$$K_1, k_3 f_{cu} b x_u = f_s A_s$$

□ — — — — —  
— — — — —

□ — — — — —

Value of  $f_s$  can be graphically computed for a given value of  $P$  as shown in Fig 2.6

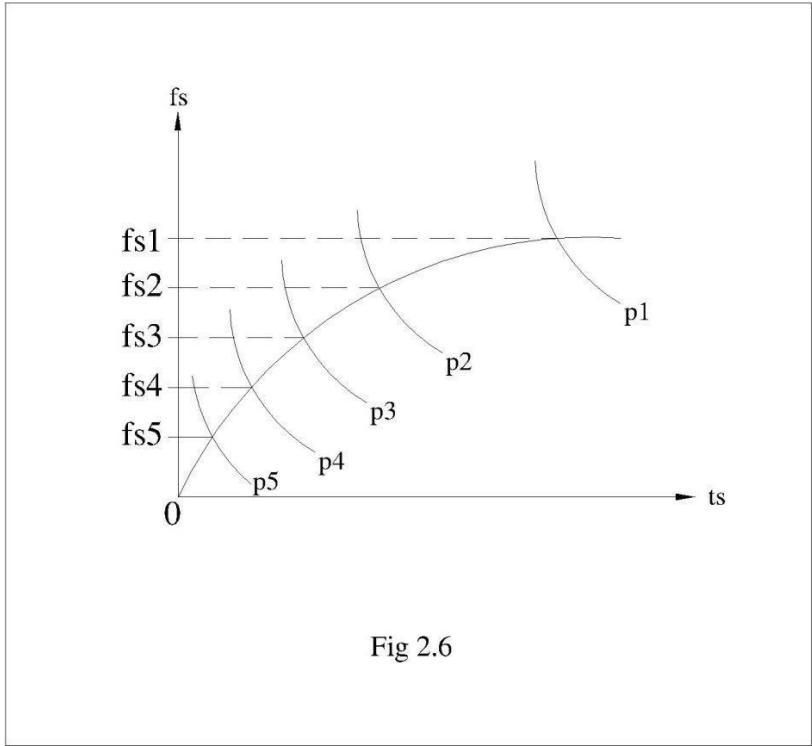


Fig 2.6



After getting  $f_s$  graphically, the ultimate moment or ultimate moment of resistance is calculated as

$$M_u = T_u \times Z = f_s A_s (d - k_2 x_u)$$

$$M_u = C_u \times Z = k_1 k_2 f_{cu} b x_u \times (d - k_2 x_u)$$

Consider

$$M_u = f_s A_s (d - k_2 x_u) = f_s A_s d (1 - k_z \frac{x_u}{d})$$

From (4) \_\_\_\_\_

$$\square \left( \frac{M_u}{f_{cu} b d^2} \right)$$

Here the term  $1 - \frac{k_2 x_u}{d}$  is called lever arm factor

Using  $A_s = p b d$  in (5), the ultimate moment of resistance is computed as

$$\left( \frac{M_u}{f_{cu} b d^2} \right) = p (1 - k_z \frac{x_u}{d})$$

\_\_\_\_\_ Dividing both sides by  $f_{cu}$  we get or

$$\frac{M_u}{f_{cu} b d^2} = p (1 - k_z \frac{x_u}{d})$$

A graph plotted between as shown in fig 2.7 and can be used for design

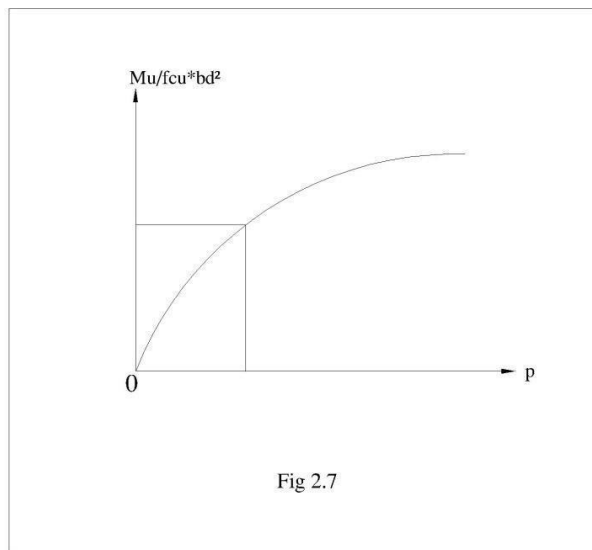


Fig 2.7

## 2.5 Stress Blocks

Stress blocks adopted by different codes are based on the stress blocks proposed by different investigators. Among them that proposed by Hog nested and Whitney equivalent rectangular block are used by most of the codes.

### 2.5.1 Stress block of IS456 – 2000

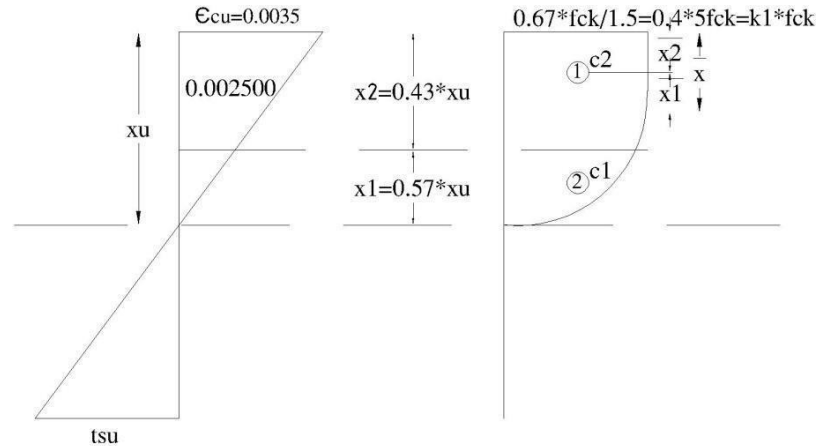


Fig 2.8

Stress block of IS456-2000 is shown in Fig 2.8. Code recommends ultimate strain  $\epsilon_{cu} = 0.0035$  & strain at which the stress reaches design strength  $\epsilon_0 = 0.002$ . Using similar triangle properties on strain diagram

□

$$\text{and } x_2 = x_u - 0.57x_u = 0.43x_u$$

Area of stress block is  $A = A_1 + A_2$ .

$A =$

$$= 0.171 f_{ck} x_u + 0.1935 f_{ck} x_u.$$

$$A = 0.3645 f_{ck} x_u \quad \square \quad (8)$$

Depth of neutral axis of stress block is obtained by taking moment of areas about extreme compression fiber.

$$\square - \frac{\Sigma}{\Sigma} \quad ( \quad ) \quad \text{---}$$

The stress block parameters thus are

$$K_1 = 0.45$$

$$K_2 = 0.42 \quad - (10)$$

$$K_3 = \text{---}$$



#### 4.5.5 Analysis of rectangular beam using IS456-2000 stress block

Case 1: Balanced section

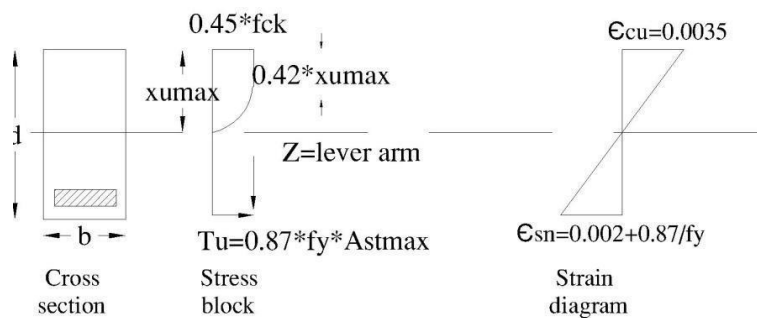


Fig 2.9

Balanced section is considered when the ultimate strain in concrete and in steel are reached simultaneously before collapse.

For equilibrium  $C_u = T_u$

$$\square 0.36f_{ck}x_{u\max} b = 0.87f_y A_{st\max}$$

$x_{u\max} = \frac{0.87f_y A_{st\max}}{0.36f_{ck}b}$  dividing both sides by

$\frac{0.87f_y A_{st\max}}{0.36f_{ck}b}$  but  $\frac{0.87f_y A_{st\max}}{0.36f_{ck}b} < p_{t\max}$ .

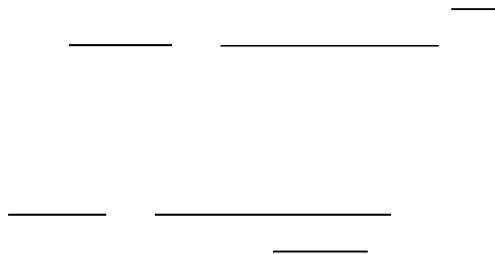
$\square$

$\left( \frac{0.87f_y A_{st\max}}{0.36f_{ck}b} \right) < p_{t\max}$

$\frac{0.87f_y A_{st\max}}{0.36f_{ck}b} < p_{t\max}$

}

From strain diagram



Values of  $\frac{0.87f_y A_{st\max}}{0.36f_{ck}b}$  is obtained from equation (12). This value depends on grade of steel. Based on grade of steel this value is given in note of clause 38.1 as (pp70)

$F_y$	$X_{u\max}/d$
250	0.53
415	0.48
500	0.46(0.456)

$p_{t\max}$  given in equation (11) is called limiting percentage steel and denoted as  $p_{t\lim}$ .

To find moment of resistance, the internal moment of  $C_u$  &  $T_u$  is computed as

$$M_{ulim} = C_u \times Z = 0.36 f_{ck} x_{ulim} b (d - 0.42 x_{ulim})$$

From equation (11) \_\_\_\_\_

$$M_{ulim} = T_u \times Z = 0.87 f_y A_{st} [d - 0.42 x_{ulim}]$$

$$M_{ulim} = 0.87 f_y A_{st} [d - 0.42 x_{ulim}]$$

$$= 0.87 f_y A_{st} [1 - \frac{0.42 x_{ulim}}{d}]$$

$$\frac{0.36 f_{ck} x_{ulim} b (d - 0.42 x_{ulim})}{0.87 f_y A_{st} [1 - \frac{0.42 x_{ulim}}{d}]}$$

From equation 4.5-5-2  $p_{lim}$  can be expressed as

$$\frac{0.36 f_{ck} x_{ulim} b (d - 0.42 x_{ulim})}{0.87 f_y A_{st} [1 - \frac{0.42 x_{ulim}}{d}]}$$

Values of \_\_\_\_\_ & \_\_\_\_\_

For different grade of Steel is given in Table (page 10 of SP -16. This table is reproduced in table 2.1.

Table 2.1 Limiting Moment resistance & limiting steel

$F_y$	250	415	500
_____	0.149	0.138	0.133
_____	21.97	19.82	18.87

Where  $p_{lim}$  is in%

Now considering  $M_{ulim} = C_u \times Z$ .

$$M_{ulim} = 0.36f_{ck}x_{ulim} b \times (d - 0.42x_{ulim})$$

\_\_\_\_\_ [ \_\_\_\_\_ ]

Value of \_\_\_\_\_ is available in table C of SP16 & Value of \_\_\_\_\_ for different grade of concrete and steel is given in Tables. Value of  $p_{lim}$  for different grade of concrete and steel is given in table E of SP-6. \_\_\_\_\_ is termed as limiting moment of resistance factor and denoted Term as  $Q_{lim}$

□  $M_{ulim} = Q_{lim}bd^2$ .

Case 2: Under reinforced section

In under reinforced section, the tensile strain in steel attains its limiting value first and at this stage the strain in extreme compressive fiber of concrete is less than limiting strain as shown in Fig 2.10

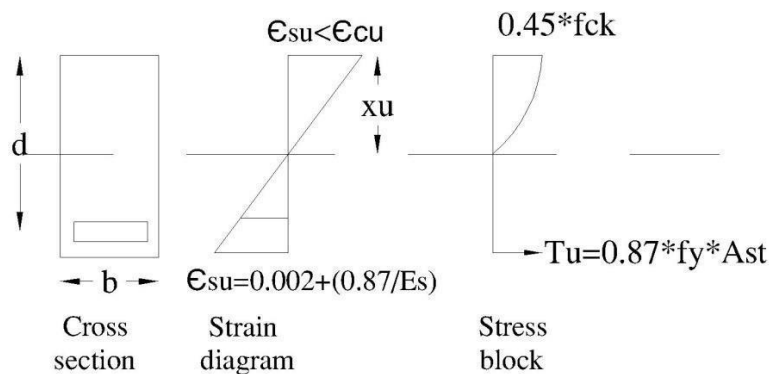


Fig 2.10

The neutral axis depth is obtained from equilibrium condition  $C_u = T_u$

$$\square 0.36f_{ck}x_{ub} = 0.87f_y A_{st}$$

$$x_u = \frac{0.87f_y A_{st}}{0.36f_{ck}b}$$

$$\text{or } \frac{x_u}{d} = \frac{0.87f_y A_{st}}{0.36f_{ck}bd}$$

Moment of resistance is calculated considering ultimate tensile strength of steel  $\square M_{uR} = T_u \times Z$   
 or  $M_{uR} = 0.87f_y A_{st} X (d - 0.42x_u)$

$$= 0.87f_y A_{st} d (1 - 0.42 \frac{x_u}{d})$$

$$= 0.87f_y A_{st} d (1 - 0.42 \times 2.41 \frac{M_{uR}}{0.87f_y A_{st} d})$$

Considering  $p_t = 100$  — expressed as % we get

$$M_{uR} = 0.87f_y A_{st} d (1 - 1.0122 \frac{M_{uR}}{0.87f_y A_{st} d})$$

$$\text{Or } \frac{M_{uR}}{0.87f_y A_{st} d} = 1 - 1.0122 \frac{M_{uR}}{0.87f_y A_{st} d}, \text{ taking } 1.0122 \approx 1$$

$$\left( \frac{M_{uR}}{0.87f_y A_{st} d} \right) \left( 1 + 1.0122 \right) = 1$$

$$\text{Or } \left( \frac{M_{uR}}{0.87f_y A_{st} d} \right) = \frac{1}{2.0122}$$

Equation (17) is quadratic equation in terms of  $(p_t/100)$

Solving for  $p_t$ , the value of  $p_t$  can be obtained as

$$P_t = 50 \left[ \frac{1}{2.0122} \pm \sqrt{\left( \frac{1}{2.0122} \right)^2 - 4 \times \left( \frac{1}{2.0122} \right) \times \left( \frac{1}{2.0122} \right)} \right]$$

$$P_t = 50 \left[ \sqrt{\left( \frac{1}{2.0122} \right)^2 - 4 \times \left( \frac{1}{2.0122} \right) \times \left( \frac{1}{2.0122} \right)} \right]$$

Let  $R_u =$

$$P_t = 50 \left[ \sqrt{\quad} \right]$$

Case 3: Over reinforced section

In over reinforced section, strain in extreme concrete fiber reaches its ultimate value. Such section fail suddenly hence code does not recommend to design over reinforced section.

Depth of neutral axis is computed using equation 4.5-6. Moment of resistance is calculated using concrete strength.

$$\square M_{uR} = C_u \times Z$$

$$= 0.36 f_{ck} x_{ub} (d - 0.42x_u) - 19$$

Position of neutral axis of 3 cases is compared in Fig. 2.11

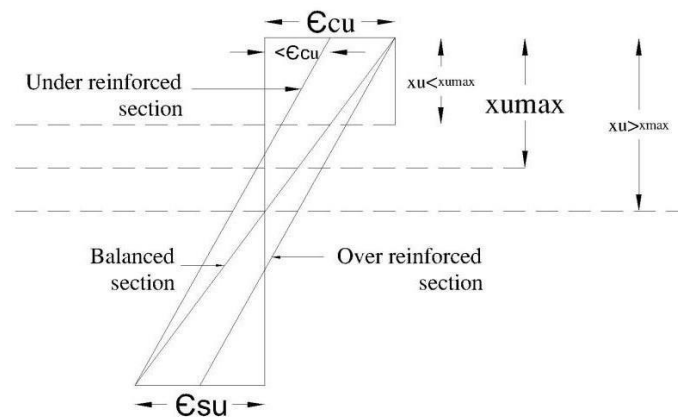


Fig 2.11



### Worked Examples

1. Determine MR of a rectangular section reinforced with a steel of area 600mm<sup>2</sup> on the tension side. The width of the beam is 200mm, effective depth 600mm. The grade of concrete is M20 & Fe250 grade steel is used.

Solve:  $f_{ck} = 20\text{Mpa}$   $f_y = 250\text{Mpa}$ ,  $A_{st} = 600\text{mm}^2$

Step . 1 To find depth of NA

— — —

$$x_u = ( \quad )^{600} = 90.375\text{mm}$$

#### Step 2 Classification

From clause 38.1, page 70 of IS456,

For Fe 250  $= 0.53$ ,  $x_{u\text{lim}} = 0.53 \times 600 = 318\text{mm}$

$x_u < x_{u\text{lim}}$ . Hence the section is under reinforced.

#### Step . 3 MR for under reinforced section.

$$\text{MR} = 0.87f_y A_{st} (d - 0.42x_u) \quad \text{—————}$$

$$\text{—————} \\ = \\ = 73.36\text{kN-m.}$$

2. Determine the MR of a rectangular section of dimension 230mm X 300mm with a clear cover of 25mm to tension reinforcement. The tension reinforcement consists of 3 bars of 20mm dia bars. Assume M20 grade concrete & Fe 415 steel.

If cover is not given, refer code – 456 → page 47

1 inch = 25mm → normal construction

Effective depth,  $d = 300 - (25 + 10)$

$$= 265\text{mm.}$$

$$A_{st} = 3 \times 20^2 = 942.48\text{mm}^2.$$

- (1) To find the depth of N.A

$$x_u = 2.41 \quad \text{—} \quad \text{—}$$

$$= 2.41$$

ii) For Fe415, —

$$x_{ulim} = 0.48 \times 442 = 212.16\text{mm}$$

$$x_u < x_{ulim}$$

The section is under reinforced.

$$\begin{aligned} \text{iii) } M_R &= 0.87f_y A_{st} (d - 0.42x_u) \\ &= 0.87 \times 415 \times 603.18 (442 - 0.42 \times 104.916) \\ &= 86.66 \text{ kN-m.} \end{aligned}$$

3. Find MR of the section with the following details.

Width of section: 230mm

Overall depth of section: 500mm

Tensile steel: 3 bars of 16mm dia

Grade of concrete: M25

Type of steel : Fe 415

Environmental condition: severe

Solve:  $b = 230$ ,  $h = 500\text{mm}$ ,  $f_{ck} = 25$ ,  $f_y = 415$

From table 16 (page 47, IS 456-2000)

Min clear cover (CC) = 45mm

Assume CC 50mm.

Effective depth =  $500 - (50 + 8) = 442\text{mm}$

$$A_{st} = 3 \times 16^2 = 603.18\text{mm}^2$$

i) To find the depth of N-A,  $x_u =$

ii) For Fe 415, = 0.48

$$x_{ulim} = 0.48 \times 300 = 144\text{mm.}$$

$x_u > x_{ulim}$  □ overreinforced.

$$\begin{aligned} (1) M_R &= (0.36f_{ck}x_u b)(d - 0.42x_u). \\ &= 60.71\text{kN-m.} \end{aligned}$$

4. A R – C beam 250mm breadth & 500mm effective depth is provided with 3 nos. of 20mm dia bars on the tension side, assuming M20 concrete & Fe 415 steel, calculate the following:

(i) N-A depth (ii) compressive force (iii) Tensile force (iv) ultimate moment (v) safe concentrated load at mid span over an effective span of 6m.

Solve:  $d=500\text{mm}$ ,  $b=250\text{mm}$

$$A_{st} = 3 \times 20^2 = 942.48\text{mm}^2$$

$$f_{ck} = 20\text{Mpa} \quad f_y = 415\text{Mpa}$$

Step – 1

$$\frac{M_u}{b d^2} = \frac{143.1 \times 10^3}{250 \times 500^2}$$

$$= 2.41$$

$$x_u = 188.52\text{mm}$$

Step – 2: For Fe 415  $\frac{x_u}{d} = 0.48$  ;  $x_{ulim} = 0.48 \times 500$

$$= 240\text{mm}$$

□  $x_u < x_{ulim}$ , the section is under reinforced.

$$C_u = 0.36 f_{ck} x_u b = 0.36 \times 20 \times 188.52 \times 250 / 10^3 = 339.34\text{kN}$$

Step – 3 MR for under reinforced section is

$$M_u = MR = 0.87 f_y A_{st} (d - 0.42 x_u)$$

$$= 143.1\text{kN-m}$$

Step – 4

$$M_u = \frac{W_u L^2}{8}$$

Equating factored moment to MR

$$\frac{W_u L^2}{8} = 143.1$$

$$W_u = 95.5\text{KN}$$

Safe load,  $W = \frac{W_u}{\text{load factor/factor of safety}}$

$$= 63.67\text{kN}$$

Step .2  $T_u = \quad \quad \quad = 340.28\text{kN}.$

$$C_u \approx T_u.$$

5. In the previous problem, determine 2 point load value to be carried in addition to its self weight, take the distance of point load as 1m.

Solve: Allowable moment, — —

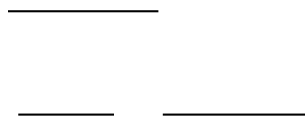
Considering self weight & the external load,

$M=MD+ML$ : MD = dead load moment, ML = live load moment, qd = self weight of beam = volume X density: density =  $25\text{kN/m}^3$  for R C C IS 875 –part – 1, plain concrete =  $29\text{kN/m}^3$

$$\text{Volume} = b \times h \times 1\text{m}$$

$$\text{Let } CC= 25\text{mm}, Ce = 25 + = 3$$

$$H= 500+35 = 535\text{mm}$$



$$M= 95.4= 15.03 + ML$$

$$ML = 80.37 \text{ kN-m}$$

$$80.37 = WLX1$$

$$WL = 80.37\text{kN}$$

6. A singly reinforced beam 200mm X 600mm is reinforced with 4 bars of 16mm dia with an effective cover of 50mm. effective span is 4m. Assuming M20 concrete & Fe 215 steel, let the central can load p that can be carried by the beam in addition to its self weight max 5m

$$= \text{---}$$

$$\text{Solve: } A_{st} = 4 \times 16^2 = 804.25\text{mm}^2$$

$$B=200\text{mm}, d=550\text{mm}, h=600\text{mm}, f_{ck} = 20\text{Mpa}, f_y = 250\text{Mpa}.$$

Step (1) — — —

$$x_u = 2.41$$

$$= 121.14 \text{ mm}$$

Step (2)

— for Fe 250 us 0.53

$$\square x_{u \text{ lim}} = 0.53 \times 550 = 291.5 \text{ mm}$$

$x_u < x_{u \text{ lim}}$  ∴ section is under reinforced.

Step – 3

$$M_u = MR = 0.87 f_y A_{st} [d - 0.42 x_u]$$

[

$$M_u = 87.308 \text{ kNm.}$$

$$M = \text{Allowable moment} = 58.20 \text{ kNm}$$

$$M = M_D + M_L$$

$q_d =$  self weight of beam

$q_d =$  volume  $\times$  density

density =  $25 \text{ kN/m}^3$  for R C C

$$\text{Volume} = b \times h \times 2 = 200 \times 600 \times 2 = 2,40,000 \text{ m}^3$$

$$q_d = 0.2 \times 0.6 \times 1 \times 25 = 3 \text{ kN/m}$$

$$q_d = \text{—————}$$

$$q_d = 6 \text{ kN/m}$$

$$M_d = \text{—————}$$

$$M_d = 6 \text{ kN.m}$$

$$M = 58.2 = 6 + M_L$$

$$M_L = 52.2 \text{ kN.m}$$

$$M_L = \frac{P \cdot L}{4}$$

$$P = \frac{4 M_L}{L}$$

$$P = 52.2 \text{ kN}$$

7. Determine N-A depth & MR of a rectangular beam of section 300mm X 600mm. The beam is reinforced with 4 bars of 25mm having an effective cover of 50mm, assume grade of concrete & steel as M20 & Fe 415 respectively

Solve:  $b=300\text{mm}$ ,  $h=600\text{mm}$ ,  $d=550\text{mm}$ .

$$f_{ck} = 20 \text{ Mpa } f_y = 415 \text{ Mpa}, A_{st} = 1963 \text{ mm}^2$$

Step- 1 Neutral axis depth

$$x_u = \frac{A_{st} f_y}{b f_{ck}}$$

$$x_u = 327.21 \text{ mm}$$

Step – 2 Classification

$$\frac{x_u}{d} = 0.48 \rightarrow x_{u\text{lim}} = 0.48 \times 550 = 264 \text{ mm}$$

$x_u > x_{u\text{lim}}$ . Hence the section is over – reinforced.

NOTE: Whenever the section is over reinforced, the strain in steel is less than the ultimate strain (0.002 +  $\frac{f_y}{E_s}$ ). Hence actual N\_A depth has to be computed, by trial and error concept because in the above equation of  $x_u$ , we have assumed the stress in steel as yield stress & this is not true.

Step – 3 Actual N-A depth (which lies b/w  $x_u = 327$  mm &  $x_{u\text{lim}} = 264$ mm)

Trial 1: Let  $x_u = 295.5$ mm

$$\frac{0.36 f_{ck} x_u b}{f_s A_{st}} = \frac{0.36 \times 20 \times 295.5 \times 1000}{351.8 + 2.36 \times 295.5 \times 1000}$$

$$= 0.00303$$

For HYSD bar, the stress strain curve is given in the code, however for definite strain values, the stress is given in SP – 16

$$\frac{0.36 f_{ck} x_u b}{f_s A_{st}} = \frac{0.36 \times 20 \times 295.5 \times 1000}{351.8 + 2.36 \times 295.5 \times 1000}$$

$$f_s = 351.8 + y^1 = 351.8 + 2.36 = 3.54.16 \text{Mpa}$$

Equating compressive force to tensile force,

$$C_u = T_u$$

$$0.36 f_{ck} x_u b = f_s A_{st}$$

$$x_u = 321.86 \text{mm}$$

Compared to the earlier computation, this value is less than 327. However to confirm we have to repeat the above procedure till consecutive values are almost same.

Trial 2 Let  $x_u = 308$ mm  $\approx 317.7$

Repeat the computation as in trial 1.

$$\frac{0.36 f_{ck} x_u b}{f_s A_{st}} = \frac{0.36 \times 20 \times 308 \times 1000}{351.8 + 2.36 \times 308 \times 1000}$$

For HYSD bars, the stress strain curve is given in the code, however for definite strain values, the stress is given in SP-16.

---


$$f_s = 342.8 + 8.74 = 351.54 \text{ Mpa}$$

Equating compressive forces to tensile forces,

$$C_u = T_u$$

$$0.36 f_{ck} b x_u = f_s A_{st}$$

$$0.36 \times 20 \times 300 \times x_u = 351.54 \times 1963.$$

$$x_u = 319.48 \text{ mm}$$

Compared to earlier computation this value is lesser than 321.8. However to confirm we have to repeat the above procedure till consecutive values are almost same.

Trial 3 Let  $x_u =$  \_\_\_\_\_ 313.74mm

---

---

$$= 0.00263$$

For HYSD bars, the stress strain curve is given in the code, however for definite strain values, the stress is given in SP-16.

---

Equating compressive forces to tensile forces,

$$C_u = T_u$$

$$0.36 f_{ck} x_{ub} = f_s A_{st}$$



$$0.36 \times 20 \times 300 \times x_u = 348.46 \times 1963.$$

$$x_u = 316.7 \text{ mm}$$

Step – 4 MR

Dia (mm)	Area
8	50
10	78.5
12	113
16	201
20	314
25	490

$$MUR = 0.36 f_{ck} x_u b (d - 0.92 x_u)$$

---


$$= 284.2 \text{ kN-m}$$

A rectangular beam 20cm wide & 40cm deep up to the center of reinforcement. Find the reinforcement required if it has to resist a moment of 40kN-m. Assume M20 concrete & Fe 415 steel.

NOTE: When ever the loading value or moment value is not mentioned as factored load, assume then to be working value. (unfactored).

Solve:  $b = 200 \text{ mm}$ ,  $d = 400 \text{ mm}$ ,  $f_{ck} = 20 \text{ Mpa}$ ,  $f_y = 415 \text{ Mpa}$

$$M = 40 \text{ kN-m}, M_u = 1.5 \times 40 = 60 \text{ kN-m} = 60 \times 10^6$$

$$M_u = MUR = 0.87 f_y \times A_{st} \times (d - 0.42 x_u) \rightarrow \textcircled{1}$$

— — —

---

Substituting in 1.

$$60 \times 10^6 = 0.87 \times 415 \times A_{st} (400 - 0.42 \times 0.25 \times A_{st})$$

$$60 \times 10^6 = 1,44,420 A_{st} - 37.91 A_{st}^2$$

$$A_{st} = 474.57 \text{ mm}^2 \text{ [ take lower value } \rightarrow \text{ under reinforced ]}$$

- In beams, dia of reinforcement is taken above 12.  
Provide 2-#16 & 1-#12

$$(A_{st})_{\text{provided}} = 2 \times 16^2 + 1 \times 12^2 = 515.2 \text{ mm}^2 > 474.57 \text{ mm}^2$$

Check for type of beam

$$\frac{M_u}{b d^2} = \dots$$

From code,  $x_{u\text{max}} = 0.48d = 192 \text{ mm} : x_u < x_{u\text{max}}$

For M20 & Fe 415

- Section is under reinforced.  
Hence its Ok . . . .

9. A rectangular beam 230mm wide & 600mm deep is subjected to a factored moment of 80kN-m. Find the reinforcement required if M20 grade concrete & Fe 415 steel is used.

Solve:  $b=230 \text{ mm}, h=600 \text{ mm}, M_u=80 \text{ kN-m}, f_{ck} = 20 \text{ Mpa}, f_y=415 \text{ Mpa}, C_e=50 \text{ mm},$   
 $= 80 \times 10^6 \text{ N-mm}$

$$M_u = M_u R = 0.87 f_y A_{st} (d - 0.42 x_u) \rightarrow \textcircled{1}$$

$$\frac{M_u}{b d^2} = \dots$$

Substituting in ①

$$80 \times 10^6 = 0.87 \times 415 \times A_{st} (0.42 \times 0.217 A_{st})$$

Procedure for design of beams

1. From basic equations.

Data required: a) Load or moment & type of support b) grade of concrete & steel.

Step . 1 If loading is given or working moment is given, calculate factored moment. ( $M_u$ )

\_\_\_\_\_

Step – 2 Balanced section parameters

$$x_{ulim}, Q_{lim}, p_{lim}, \text{ (table A to ESP 16)} \quad Q_{lim} = \text{---}$$

Step – 3 Assume b and find

$$d_{lim} = \sqrt{\text{---}}$$

Round off  $d_{lim}$  to next integer no.

$h = d + C_e$  :  $C_e$  = effective cover : assume  $C_e$ .

$C_e = 25\text{mm}$ :  $C_e = C_c + \text{---}$

Step – 4 To determine steel.

Find  $P_t =$  \_\_\_\_\_

$A_{st} =$  : assume suitable diameter of bar & find out no. of bars required.

$N = \frac{A_{st}}{a_{st}}$  = area of 1 bar.

Using design aid SP16

Step 1 & Step 2 are same as in the previous case i.e using basic equation

Step – 3 Find and obtain  $p_t$  from table 1 to 4 → page 47 – 50

Which depends on grade of concrete.

From  $p_t$ , calculate  $A_{st}$  as  $A_{st} =$  \_\_\_\_\_

Assuming suitable diameter of the bar, find the no. of re-bar as  $N =$  \_\_\_\_\_

where  $Q_{st} =$

NOTE: 1 . To find the overall depth of the beam, use clear cover given in IS 456 – page 47 from durability & fire resistance criteria.

2. To take care of avoiding spalling of concrete & unfavourable tensile stress, min. steel has to be provided as given in IS 456 – page 47 — \_\_\_\_\_

If  $A_{st}$  calculated, either by method 1 or 2 should not be less than  $(A_s)_{min}$  .

If  $A_{st} < A_{smin}$  :  $A_{st} = A_{smin}$

1. Design a rectangular beam to resist a moment of 60kN-m, take concrete grade as M20 & Fe 415 steel.

Solve:  $M = 60$  KN-m

$$M_u = 1.5 \times 60 = 90 \text{ kN-m}$$

$$f_{ck} = 20 \text{ Mpa}, f_y = 415 \text{ Mpa}$$

Step 1: Limiting Design constants for M<sub>20</sub> concrete & Fe 415 steel.

———— = 0.48 : From Table – c of SP-16, page 10

$$Q_{lim} = 2.76$$

$$P_{t \text{ lim}} = 0.96$$

column sizes 8 inches = 200mm

9 inches = 230mm

Step - 2 :  $\sqrt{\text{————}}$

Let b = 230mm

$$\sqrt{\text{————}}$$

Referring to table 16 & 16 A, for moderate exposure & 1½ hour fire resistance, let us assume clear cover C<sub>c</sub>=30mm & also assume 16mm dia bar □ effective cover C<sub>e</sub> = 30 + 8 = 38mm

$$h_{bal} = 376.5 + 38 = 414.5$$

18 inches = 450mm

Provide overall depth, h= 450mm.

„d“ provided is 450-38 = 412mm

Step .3 Longitudinal steel

Method . 1 → using fundamental equations

Let p<sub>t</sub> be the % of steel required

$$[\sqrt{\text{————}}] = 0.53$$

$$= [\sqrt{\text{————}}]$$

$$= 0.758$$

Method .2 → using SP 16. Table – 2 page – 47 use this if it is not Specified in problem

$$K=2.3 \rightarrow pt = 0.757$$

$$K= 2.32 \rightarrow pt = 0.765$$

$$\text{For } k = 2.305, pt= 0.757+$$

$$= 0.759$$

Step 4 : detailing

$$\text{Area of steel required, } A_{st} = \text{---} = 720\text{mm}^2$$

- M20  $\rightarrow$  combination 12mm & 20mm aggregate (As) size.
- Provide 2 bars of 20mm & 1 bar of 12mm,
- $\square$  Ast provided =  $2 \times \frac{3}{4} + 113$   
=  $741 > 720\text{mm}^2$

[ To allow the concrete flow in b/w the bars, spacer bar is provided]

1.Design a rectangular beam to support live load of 8kN/m & dead load in addition to its self weight as 20kN/m. The beam is simply supported over a span of 5m. Adopt M25 concrete & Fe 500 steel. Sketch the details of c/s of the beam.

$$\text{Solve: } q_L=8\text{kN/m } b=230\text{mm } = 20\text{kN/m } f_{ck} = 25\text{Mpa } f_y = 500\text{Mpa. } l= 5\text{m}=5000\text{mm}$$

Step .1: c/s

NOTE; The depth of the beam is generally assumed to start with based on deflection criteria of serviceability. For this IS 456-2000 – page 37, clause 23.2-1 gives = 20

with some correction factors. However for safe design generally l/d is taken as 12

$$\text{---} = 416\text{mm. (no decimal)}$$

$$\text{Let } C_e = 50\text{mm, } h=416+50 = 466\text{mm}$$

### Step 2 Load calculation

i) Self weight =  $0.23 \times 0.5 \times 1 \times 25 = 2.875 \text{ kN/m} =$

ii) dead load given = 20kN

qd =  $\quad + \quad = 22.875 \text{ kN/m} \quad 25 \text{ kN/m}$  [multiple of 5]

[Take dead load as x inclusive of dead load, don't mention step . 2]

iii) Live load = 8kN/m

$\quad \quad \quad = 78 \text{ kN-m}$  (no decimal)

Dead load  $\quad \quad$  Live load moment =  $\quad \quad$

$M_u = 1.5MD + 1.5ML = 1.5 \times 78 + 1.5 \times 25 = 154.5 \text{ kN-m}$

### Step -3 Check for depth

$d_{bal} = \sqrt{\quad}$

From table.3,  $Q_{lim} = 3.33$  (page – 10) SP-16

( $h_{bal} < h$  assumed condition for Safe design)

$\quad \quad \quad$   
 $\quad \quad \quad$

$H_{bal} = 449.13 + 50 = 499.13$

Hence assumed overall depth of 500mm can be adopted.

Let us assume 20mm dia bar &  $C_c = 30 \text{ mm}$  (moderate exposure & 1.5 hour fire resistance)

$\square$   $C_e$  provided =  $30 + 10 = 40$ ,  $d_{provided} = 500 - 40 = 460 \text{ mm}$

### Step .4 Longitudinal steel

Page 49 – SP-16

$M_u/bd^2$  pt

3.15 0.880  
3.20 0.898

For

$$= 0.8872$$

No. of # 20 =

$$(A_{st})_{provided} = 3 \times 314 = 942\text{mm}^2 > 938.66\text{mm}^2.$$

Page – 47 – IS456

$$(A_{st})_{min} =$$

$$\square (A_{st})_{provided} > (A_{st})_{min} . \text{ Hence o.k.}$$

Step . 5 Detailing

3. Design a rectangular beam to support a live load of 50 kN at the free end of a cantilever beam of span 2m. The beam carries a dead load of 10kN/m in addition to its self weight. Adopt M30 concrete & Fe 500 steel.

$$l=2\text{m}=2000\text{mm}, =10, f_{ck}= 30\text{Mpa}, f_y=500\text{Mpa}, qL=50\text{kN}, b=230\text{mm}$$

Step – 1 c/s



NOTE: The depth of the beam is generally assumed to start with based on deflection criteria of serviceability. For this IS 456-2000-Page – 37, clause 23.2.1 gives

$= 5$  with some correction factor.

$d = 400 \text{ mm}$

Let  $C_c = 50 \text{ mm}$ ,  $h = 400 + 50 = 450 \text{ mm}$

However we shall assume  $h = 500 \text{ mm}$

Step – 2 Load calculation.

(i) Self wt =  $0.23 \times 0.5 \times 1 \times 25 = 2.875 \text{ kN/m}$

(ii) Dead load given =  $10 \text{ kN/m}$

$q_d = 10 + 2.875 = 12.875 \text{ kN/m} \approx 15 \text{ kN/m}$  [multiply of 5]

(iii) Live load =  $50 \text{ kN}$

$$M_L = W \times L = 50 \times 2 = 100 \text{ kN-m}$$

$$M_u = 1.5M_D + 1.5M_L = 195 \text{ kN-m}$$

Step – 3 Check for depth

$d_{bal} = \sqrt{\frac{M_u}{Q_{lim}}} = \sqrt{\frac{195}{3.99}} = 7.01 \text{ m}$

$$d_{bal} = \sqrt{\frac{195}{3.99}} = 7.01 \text{ m}$$

$h_{bal} = 7.01 + 0.5 = 7.51 \text{ m}$

hassumed =  $550 \text{ mm}$ .

Let us assume  $20 \text{ mm}$  dia bars &  $C_c = 30 \text{ mm}$  constant

□  $C_e \text{ provided} = 30 + 10 = 40$ ,  $d_{\text{provided}} = 550 - 40 = 510 \text{ mm}$

Step – 4 Longitudinal steel

Page – 49m SP-16m

$M_u/bd^2$	pt
3.25	0.916
3.30	0.935

For  $M_u/bd^2 = 3.26$ ,

$$P_t = 0.916 + \frac{3.26 - 3.25}{3.30 - 3.25} (0.935 - 0.916)$$

$$= 0.9198.$$

$$A_{st} = \frac{M_u}{P_t \sigma_{st} d} = 1078.9 \text{ mm}^2$$

$$3\text{-}\#20, 1\text{-}\#16 = 1143$$

$$2\text{-}\#25, 2\text{-}\#20 = 1382$$

$$(A_{st})_{\text{Provided}} =$$

$$(A_{st})_{\text{min}} =$$

Step – 5 Detailing

Design of slabs supported on two edges

Slab is a 2 dimensional member provided as floor or roof which directly supports the loads in buildings or bridges.

In RCC, it is reinforced with small dia bars (6mm to 16mm) spaced equally.

Reinforcement provided in no. RCC beam → 1 dia width is very (12mm – 50mm) small compared to length. Element with const. Width: fixed width.

It is subjected to vol.

RCC slabs → reinforcement are provided with equally spaced. No fixed width & length are comparable, dia -6mm to 16mm. It is subjected to pressure.

Beams are fixed but slabs are not fixed. For design, slab is considered as a beam as a singly reinforced beam of width 1m

Such slabs are designed as a beam of width 1m & the thickness ranges from 100mm to 300mm.

IS456-2000 stipulates that for simply supported slabs be 35 & for continuous slab 40 (page – 39). For calculating area of steel in 1m width following procedure may be followed.

$$A_{st} = N \quad \text{—————} \quad \text{—————}$$

For every 10cm there is a bar

$$S = \text{—————}$$

(The loading on the slab is in the form of pr. expressed as kN/m<sup>2</sup>)

As per clause 26.5.2-1 (page 48 min steel required is 0-15% for mild steel & 0-12% for high strength steel. It also states that max. dia of bar to be used is 1/8<sup>th</sup> thickness of the slab. To calculate % of steel we have to consider gross area is 1000Xh.

The slabs are subjected to low intensity secondary moment in the plane parallel to the span. To resist this moment & stresses due to shrinkage & temp, steel reinforcement parallel to the span is provided. This steel is called as distribution steel. Min. steel to be provided for distribution steel.

Practically it is impossible to construct the  
Slab as simply supported bozo of partial bond

b/w masonry & concrete, also due to the parapet wall constructed above the roof slab. This induces small intensity of hogging BM. Which requires min. % of steel in both the direction at the top of the slab as shown in fig. 2 different types of detailing is shown in fig.

Method – 1

Crank → for the change in reinforcement.

(1) Alternate cranking bars.

Dist. Steel is provided.

- To take care of secondary moment, shrinkage stresses & temp. stress steel is provided parallel to the span.

Method – 2

1. Compute moment of resistance of a 1- way slab of thickness 150mm. The slab is reinforced with 10mm dia bars at 200mm c/c. Adopt M20 concrete & Fe 415 steel. Assume  $C_e=20\text{mm}$ .

Solve:  $h=150\text{mm}$ ,  $C_e=20\text{mm}$ ,  $\phi=10\text{mm}$

$$d=150-20 =130\text{mm}, s_x=200\text{mm}$$

$$A_{st} = \frac{M}{f_y \times d} \times \frac{100}{\rho}$$

$$= \frac{M}{f_y \times d} \times \frac{100}{\rho}$$

$$= 392.7\text{mm}^2$$

Step - 1 N-A depth

— — —

$$x_u = 2.41$$

$$= 19.64 \text{ mm.}$$

$$x_{u\max} = 0.48d = 0.48 \times 130 = 62.4 \text{ mm.}$$

Step – 2 Moment of resistance

$x_u < x_{u\max}$ . hence section is under reinforced.

$$M_u R = 0.87 f_y A_{st} (d - 0.42 x_u)$$

\_\_\_\_\_

$$= 17.26 \text{ kN-m/m.}$$

Doubly reinforced Beams.

Limiting state or Balanced section.

$$C_{uc} = 0.36 f_{ck} b x_{ulim}$$

$$T_{u1} = 0.87 f_y A_{st}$$

$$M_{ulim} = 0.36 f_{ck} b x_{ulim} (d - 0.42 x_{ulim})$$

$$p_{lim} = \text{— — —}$$

$$A_{st} = \frac{M_u}{f_y Z_2}$$

$M_u > M_{u,lim}$                        $M_u =$  applied factored moment.

$$M_{u2} = M_u - M_{u,lim}$$

For  $M_{u2}$  we require  $A_{st}$  in compression zone &  $A_{st2}$  in tension zone for equilibrium

$$C_{us} = T_{u2}$$

$C_{us} = f_{sc} \cdot A_{sc}$  :  $f_{sc}$  is obtained from stress – strain curve of corresponding steel.

In case of mild steel, it is  $f_{sc} = f_y$

In case of high strength deformable bars,

$f_{sc}$  corresponding to strain  $\epsilon_{sc}$  should be obtained from table A-SP-16 (Page – 6)

$$Z_2 = d - d^1 \quad \text{If } \epsilon_{sc} < 0.00109,$$

$$f_{sc} = \epsilon_s X \epsilon_{sc}$$

$$\text{else } f_{sc} = f_y / 1.15$$

$$T_{u2} = 0.87 f_y A_{st2} \leq 1$$

$$\text{Couple } M_{u2} = C_{us} X Z_2 = T_{u2} Z_2$$

$$M_{u2} = (f_{sc} A_{sc})(d - d^1)$$

$$C_{us} = T_{u2}$$

$$f_{sc} X A_{sc} = 0.87 f_y A_{st2}$$

Procedure for design of doubly reinforced section.

Step. 1 Check for requirement of doubly reinforced section

1. Find  $x_{ulim}$  using IS456
2. Find  $M_{ulim}$  for the given section as  $M_{ulim} = Q_{lim} X b d^2$  Refer table – D, page – 10, of SP – 16 for  $Q_{lim}$
3. Find  $p_{tlim}$  from table – E page 10 of SP – 16 & then compute.

Step – 2 If  $M_u > M_{ulim}$  then design the section as doubly reinforced section, else design as singly reinforced section.

Step – 3  $M_{u2} = M_u - M_{ulim}$

Step – 4 Find area of steel in compression zone using the equation as

$A_{sc} =$  \_\_\_\_\_  $f_{sc}$  has to be obtained from stress – strain curve or from table – A, page – 6 of SP-16.

The strain  $\epsilon_{sc}$  is calculated as \_\_\_\_\_

Step - 5 Additional tension steel required is computed as

□ Total steel required  $A_{st} = A_{st1} + A_{st2}$  in tension zone.

Use of SP-16 for design of doubly reinforced section table – 45-56, page 81-92 provides  $p_t$  &  $p_c$ :

$p_t =$  \_\_\_\_\_

$p_c =$  \_\_\_\_\_ X100 for different values of  $M_u/bd^2$  corresponding to combination of  $f_{ck}$  &  $f_y$ .

Following procedure may be followed.

Step – 1 Same as previous procedure.



Step – 2 If  $M_u > M_{u\text{lim}}$ , find  $M_u/bd^2$  using corresponding table for given  $f_y$  &  $f_{ck}$  obtain  $p_t$  &  $p_c$ . Table – 46.

NOTE: An alternative procedure can be followed for finding  $f_{sc}$  in case of HYSD bars i.e use table – F, this table provides  $f_{sc}$  for different ratios of  $d'/d$  corresponding to Fe 415 & Fe 500 steel

Procedure for analysis of doubly reinforced beam

Data required:  $b, d, d', A_{st}(A_{st1} + A_{st2}), A_{sc}, f_{ck}, f_y$

Step – 1 Neutral axis depth

$$C_{uc} + C_{us} = T_u$$

$$0.36f_{ck} < b x_u + f_{sc} A_{sc} = 0.87f_y A_{st}$$

$$x_u = \text{—————}$$

This is approximate value as we have assumed the tensile stress in tension steel is  $0.87f_y$  which may not be true. Hence an exact analysis has to be done by trial & error. (This will be demonstrated through example).

Step – 2: Using the exact analysis for N-A depth the MR can be found as.

$$M_{u\text{lim}} = 0.36f_{ck} \cdot b x_u (d - 0.42 x_u) + f_{st} A_{sc} X (d^2 - d^1)$$

1. Design a doubly reinforced section for the following data.

$$b = 250\text{mm}, d = 500\text{mm}, d' = 50\text{mm}, M_u = 500\text{kN-m con-}, M_{30}, \text{steel} = \text{Fe 500.}$$

$$= 0.1, f_{ck} = 30\text{Mpa}, f_y = 500\text{Mpa.}$$

$$M_u = 500 \times 10^6 \text{ N-mm}$$

Step – 1 Moment of singly reinforcement section.

$$\text{Page – 70 – IS-456} = 0.46$$

$$x_{u\text{lim}} = 0.46 \times 500 = 230\text{mm.}$$

$$M_{ulim} = Q_{lim} \times bd^2 = \text{—————} = 249\text{kN-m.}$$

Page – 10 SP – 16

$$P_{lim} = 1.13: A_{st} = A_{stlim} = 1412.5\text{mm}^2$$

$$M_u > M_{ulim}$$

$$\text{Step – 2 } M_{u2} = M_u - M_{ulim} = 500 - 249 = 251\text{kN-m.}$$

$$A_{sc} = \text{—————} \text{ page – 13. SP-16. Table – F}$$

$$= \text{—————} = 1353.83\text{mm}^2$$

From equilibrium condition,

$$A_{st} = A_{st1} + A_{st2} = 1412.5 + 1282.25 = 2694.75\text{mm}^2$$

Step – 3: Detailing.

$$A_{sc} = 1353\text{mm}^2$$

$$A_{st} = 2694\text{mm}^2$$

Tension steel

$$\text{Assume \# 20 bars } = 8.5$$

However provide 2 - #25 + 6 - #20

$$(A_{st})_{provided} = 2 \times 490 + 6 \times 314 = 2864\text{mm}^2 > 2694\text{mm}^2$$

Compression steel

$$\text{Assume \#25, No } = 2.7$$

$$\text{Provide 3 - \#25 } (A_{st})_{provided} = 3 \times 490 = 1470\text{mm}^2 > 1353\text{mm}^2$$

$$C_e = 30 + 25 + 12.5 = 67.5\text{mm}$$

2. Design a rectangular beam of width 300mm & depth is restricted to 750mm(h) with a effective cover of 75mm. The beam is simply supported over a span of 5m. The beam is subjected to central con. Load of 80kN in addition to its self wt. Adopt M30 concrete & Fe 415 steel.

$$W_d = 0.3 \times 0.75 \times 1 \times 25 = 5.625$$

$$M_D = 17.6\text{kN-m}$$

$$M_L = 100 \text{ kN-m}$$

$$M_u = 1.5(M_D + M_L) = 176.4$$

3. Determine areas of compression steel & moment of resistance for a doubly reinforced rectangular beam with following data.

$b = 250 \text{ mm}$ ,  $d = 500 \text{ mm}$ ,  $d' = 50 \text{ mm}$ ,  $A_{st} = 1800 \text{ mm}^2$ ,  $f_{ck} = 20 \text{ Mpa}$ ,  $f_y = 415 \text{ Mpa}$ . Do not neglect the effect of

Compression reinforcement for calculating Compressive force.

Solve:  $C_{c1} \rightarrow$  introduce a negative force

$$\text{Note: } C_u = C_{sc} + C_c - C_{c1}$$

$$= f_{sc} A_{sc} + 0.36 f_{ck} b x_u - 0.45 f_{ck} X A_{sc}$$

$$= 0.36 f_{ck} b x_u + A_{sc} (f_{sc} - 0.45 f_{ck})$$

For calculating compressive force then,

Whenever the effect of compression steel is to be considered.

Step – 1 Depth of N-A.

From IS – 456 for M20 concrete and Fe 415 steel is

$$\text{————} = 0.48 \text{ \& } x_{u\text{max}} = 0.48 \times 500 = 240 \text{ mm}$$

From table – 6 , SP – 16 page- 10,  $p_{lim} = 0.96$

$$A_{st1} = A_{stlim} \text{ —————} = 1200 \text{ mm}^2$$

$$A_{st2} = A_{st} - A_{st1} = 1800 - 1200 = 600 \text{ mm}^2$$

Step – 2 :  $A_{sc}$

For equilibrium,  $C_u = T_u$ .

In the imaginary section shown in fig.

$$C_{u1} = A_{sc}(f_{sc} - 0.45f_{ck})$$

$$= 0.1, \text{ Table - F, SP - 16, P - 13, } f_{sc} = 353 \text{ Mpa}$$

$$\begin{aligned} A_{sc}(353 - 0.45 \times 20) &= 600 \times 0.87 \times 415 \\ A_{sc} &= 629 \text{ mm}^2 \end{aligned}$$

$$\text{Step - 3 MR} \quad A_{sc}(f_{sc} - 0.45f_{ck}) (d - d^1)$$

$$\begin{aligned} M_{ur2} &= C_{u1} \times Z_2 = \text{—————} \\ &= 97.6 \text{ kN-m} \end{aligned}$$

$$\begin{aligned} M_{ur1} = M_{ulim} &= Q_{lim} b d^2 \quad Q_{lim} = 2.76 \\ &= \text{—}^2 \text{—} = 172 \text{ kN-m} \end{aligned}$$

$$M_{ur} = M_{ur1} + M_{ur2} = 97.6 + 172 = 269.8 \text{ kN-m}$$

$$A_{sc} = 629 \text{ mm}^2, M_{ur} = 269.8 \text{ kN-m}$$

4. A rectangular beam of width 300mm & effective depth 550mm is reinforced with steel of area 3054mm<sup>2</sup> on tension side and 982mm<sup>2</sup> on compression side, with an effective cover of 50mm. Let MR at ultimate of this beam is M20 concrete and Fe 415 steel are used. Consider the effect of compression reinforcement in calculating compressive force. Use 1<sup>st</sup> principles No. SP -16.

Solve: Step: 1 N-A depth

$$\text{————} = 0.48$$

$$x_{ulim} = 0.48 \times 550 = 264 \text{ mm}$$

Assuming to start with  $f_{sc} = 0.87f_y = 0.87 \times 415 = 361 \text{ Mpa}$ .

Equating the total compressive force to tensile force we get,

$$C_u = T_u = C_u = 0.36f_{ck} b x_u + A_{st}(f_{sc} - 0.45f_{ck})$$

$$T_u = 0.87f_y A_{st}$$

$$x_u = \frac{m f_{sc}}{f_{st} - m f_{sc}} \rightarrow 1$$

$$= 350.45 \text{ mm} > x_{u\text{lim}}$$

Hence the section is over reinforced.

The exact N-A depth is required to be found by trial & error using strain compatibility, for which we use equation ① in which the value of  $f_{sc}$  is unknown, hence we get,

$$x_u = \frac{m f_{sc}}{f_{st} - m f_{sc}}$$

$$\frac{350.45}{x_u} = \frac{m f_{sc}}{f_{st} - m f_{sc}} \rightarrow \text{②}$$

Range of  $x_u$  & 264 to 350.4

$$\text{Cycle - 1 Try } (x_u)_1 = 307 \text{ mm}$$

From strain diagram  $\epsilon_{sc} = 0.0035(1 - \frac{x_u}{x_{u\text{lim}}}) = 0.00293$  (similar triangle)

$$\epsilon_{st} = 0.00279$$

$f_{sc} = 339.3 \text{ mm}$ . The difference b/w  $(x_u)_1$  &  $(x_u)_2$  is large, hence continue cycle 2.

$$\text{Cycle - 2 Try } (x_u)_3 = 323 \text{ mm}$$

From strain diagram,  $\epsilon_{sc} = 0.0035(1 - \frac{x_u}{x_{u\text{lim}}}) = 0.00296$ .

$$\epsilon_{st} = 0.00246$$

$$f_{sc} = 353.5, f_{st} = 344.1, (x_u)_4 = 328$$

The trial procedure is covering, we shall do 1 more cycle.

$$\text{Cycle - 3 Try } (x_u)_5 = 325.5 \text{ mm}$$

From strain diagram,  $\epsilon_{sc} = 0.0035(1 - \frac{x_u}{x_{u\text{lim}}}) = 0.00296$ .

$$\epsilon_{st} = 0.00241$$

$$f_{sc} = 353.5, f_{st} = 342.8, (x_u)_6 = 326.2$$

$$\square x_u = 326.2 \text{ mm}$$

Step . 2

$$M_{ur} = \frac{1}{100} \times 46300 = 463 \text{ kN-m}$$

5. Repeat the above problem for Fe 450.

Note:  $x_u < x_{u\max}$ , hence cyclic procedure is not required.

$$x_u = 211.5, M_{ur} = 315 \text{ kN-m.}$$

6. A rectangular beam of width 300mm & effective depth of 650mm is doubly reinforced with effective cover  $d' = 45\text{mm}$ . Area of tension steel 1964, area of compression steel = 982mm<sup>2</sup>. Let ultimate MR if M- 20 concrete and Fe 415 steel are used.

$$\text{Ans: } x_u = 117.34 \text{ mm, } M_{ur} = 422 \text{ kN-m}$$

Flanged sections.



Concrete slab & concrete beam are

Cast together → Monolithic construction

- Beam → tension zone, slab in comp. zone
- Slab on either side → T beam

Slab on one side → L beam

- $b_f \rightarrow$  effective width  $b_f > b$

T- beam



$$b_f = b_w + 6D_f$$

- $l_o = 0.7l_e$ : continuous & frames beam.
- A & B points of contra flexure (point of zero moment)

L-Beam

$$b_f = b_w + 3D_f$$

Isolated T- beam

It is subjected to torsion & BM

- If beam is resting on another beam it can be called as L – beam.
- If beam is resting on column it cannot be called as L- beam. It becomes –ve beam.

Analysis of T – beam :- All 3 cases NA is computed from  $C_u = T_u$ .

Case-1 – neutral axis lies in flange.

Case (ii) : NA in the web &  $\leq 0.2$

Whitney equivalent rectangular stress block.

Case (iii): NA lies in web &  $> 0.2$

1. All three cases NA is computed from  $C_u = T_u$ .
2.  $x_{u\text{lim}}$  same as in rectangular section.
  - Depth of NA for balanced s/n depends on grade of steel.
3. Moment of resistance

$$\text{Case . (1) } C_u = 0.36f_{ck}b_f x_u \quad T_u = 0.87f_y A_{st}$$

$$M_{ur} = 0.36f_{ck} b_f x_u(d-0.42x_u) \text{ or } 0.87f_y A_{st}(d - 0.42x_u)$$

Case – (ii)  $C_u = 0.36f_{ck}b_w x_u + 0.45f_{ck}(b_f - b_w)D_f (d -$

$$T_u = 0.87f_y A_{st}$$

$$M_{ur} = 0.36f_{ck} b_w x_u(d - 0.42x_u) + 0.45f_{ck}(b_f - b_w)D_f (d - )$$

Case (iii)

$$C_u = 0.36f_{ck} b_w x_u + 0.45f_{ck}(b_f - b_w)y_f.$$

$$T_u = 0.87f_y A_{st}.$$

Page . 97,  $M_{ur} = 0.36f_{ck}b_w x_u(d-0.42x_u) + 0.45f_{ck}(b_f - b_w) y_f (d - )$

Where,  $y_f = 0.15 x_u + 0.65D_f$

Obtained by equating areas of stress block.



- When  $D_f/x_u \leq 0.43$  &  $D_f/x_u > 0.43$  for the balanced section & over reinforced section use  $x_{u\max}$  instead of  $x_u$ .

Problem.

1. Determine the MR of a T – beam having following data.
  - a) flange width = 1000mm =  $b_f$
  - b) Width of web = 300mm =  $b_w$
  - c) Effective depth = ,  $d= 450$ mm
  - d) Effective cover = 50mm
  - e)  $A_{st} = 1963\text{mm}^2$
  - f) Adopt M20 concrete & Fe 415 steel.

Solve: Note:

In the analysis of T – beam, assume N-A To lie in flange & obtain the value of  $x_u$ . If  $x_u > D_f$  then analyses as case (2) or (3) depending on the ratio of  $D_f/d$ . If  $D_f/d \leq 0.2$  case (2) or  $D_f/d > 0.2$  case (3)

Step – 1

Assume NA in flange

$$C_u = 0.36f_{ck} b_f x_u \quad T_u = 0.87f_y A_{st}$$

$$C_u = T_u.$$

$$0.36f_{ck} b_f x_u = 0.87f_y A_{st}$$

$$x_u = \text{—————}$$

$$= 98.4\text{mm} < D_f = 1000\text{mm}.$$

Assumed NA position is correct i.e(case . 1)

$$\text{Step – 2 : } M_{ur} = 0.36f_{ck} b_f x_u (d - 0.42x_u)$$

$$= 0.36 \times 20 \times 1000 \times 98.4(450 - 0.42 \times 98.4)/10^6$$

$$= 290 \text{ kN-m}$$

$$\text{Or } M_{ur} = 0.87f_y A_{st} (d - 0.42x_u)$$

$$= 0.87 \times 415 \times 1963(450 - 0.42 \times 98.4)$$

$$= 290 \text{ kN-m}$$

Use of SP 16 for analysis p: 93 - 95

For steel of grade Fe 250, Fe 415 & Fe 500, SP - 16 provides the ratio  $\frac{K_T}{2}$  for combinations

of and using this table the moment of resistance can be calculated as  $M_u = K_T f_{ck} b_w d^2$  where  $K_T$  is obtained from SP -16.

Solve:  $= 0.22 > 0.2$

$$\frac{—}{—} = 3.33$$

For Fe 415m P:94

$$\frac{—}{—} = \frac{3}{4}$$

$$K_T = 0.309 \text{ } 0.395$$

$$K_T \text{ for } = 3.3 \times 0.309 + \frac{—}{—} \times (3.3-3) = 0.337$$

$$M_{ulim} = \frac{2}{—} = 410 \text{ kN-m.}$$

This value corresponds to limiting value. The actual moment of resistance depends on quantity of steel used.

2. Determine area of steel required & moment of resistance corresponding to balanced section of a T - beam with the following data,  $b_f = 1000$ ,  $D_f = 100\text{mm}$ ,  $b_w = 300\text{mm}$ , effective cover =  $50\text{mm}$ ,  $d = 450\text{mm}$ , Adopt M20 concrete & Fe 415 steel.

Use 1<sup>st</sup> principles.

Solve: Step -1  $= 0.22 > 0.2$  case (iii)

$$\text{Step - 2 } y_f = 0.15 x_{u\max} + 0.65D_f$$

$$C_u = 0.36f_{ck} b_w x_{u\max} + 0.45f_{ck}(b_f - b_w)y_f.$$

$$T_u = 0.87f_y A_{stlim}$$

For Fe 415,  $x_{u\max} = 0.48d = 216\text{mm} > D_f$ .

$$C_u = 0.36 \times 20 \times 300 \times 216 + 0.45 \times 20(1000 - 300)97.4 = 1.0801 \times 10^6 \text{Nmm}$$

$$Y_f = 0.15 \times 216 + 0.65 \times 100 = 97.4 \text{mm}$$

$$C_u = T_u$$

$$C_u = 0.87 \times 415 A_{st}$$

$$1.0801 \times 10^6 = 0.87 \times 415 A_{stim}$$

$$A_{stim} = 2991.7 \text{mm}^2$$

$$\text{Step - 3 } M_{ur} = 0.36f_{ck} b_w x_{u\max}(d - 0.42x_{u\max}) + 0.45f_{ck}(b_f - b_w)y_f(d - )$$

$$= 0.36 \times 20 \times 300 \times 216 (450 - 0.42 \times 216) + 0.45 \times 20 (1000 - 300) 97.4 (450 -$$

—)

$$M_{ur} = 413.27 \text{kN-m}$$

3. Determine M R for the c/s of previous beam having area of steel as 2591mm<sup>2</sup>

Step - 1 Assume N-A in flange

$$C_u = 0.36f_{ck}b_f x_u$$

$$T_u = 0.87 f_y A_{st}$$

For equilibrium  $C_u = T_u$

$$0.36 \times 20 \times 1000 x_u = 0.87 \times 415 \times 2591$$

$$x_u = 129.9 \text{mm} > D_f$$

□ N-A lies in web

$$— \quad \frac{x_u}{D_f} = 0.22 > 0.2 \quad \text{case (iii)}$$

$$\square y_f = 0.15x_u + 0.65D_f = 0.15 \times x_u + 0.65 \times 100 = 0.15 x_u + 65$$

$$C_u = T_u$$

$$C_u = 0.36 f_{ck} b_w x_u + 0.45f_{ck} (b_f - b_w)y_f =$$

$$T_u = 0.87f_y A_{st} = 0.87 \times 450 \times 2591 = 1014376.5$$

$$1014376.5 = 0.36 \times 20 \times 300 x_u + 0.45 \times 20(1000 - 300)84.485$$

$$x_u = 169.398\text{mm} < x_{u\text{max}} = 0.48 \times 450 = 216\text{mm}$$

It is under reinforced section.

Step – 2 MR for under reinforced section depends on following.

$$(1) = 0.22 > 0.2 \text{ (case iii)}$$

$$(2) = \frac{M_u}{b d^2} = 0.59 > 0.43$$

Use  $y_f$  instead of  $D_f$  in computation of MR

$$\square y_f = 0.15x_u + 0.65D_f$$

$$= 0.15 \times 169.398 + 0.65 \times 100$$

$$= 90.409\text{mm.}$$

$$M_{ur} = 0.36f_{ck} x_u b_w (d - 0.42x_u) + 0.45f_{ck} (b_f - b_w) y_f (d - )$$

$$= 0.36 \times 20 \times 169.398 \times 300 (450 - 0.42 \times 169.398) + 0.45 \times 20 (1000 - 300) 90.409 (450 - )$$

$$= M_{ur} = 369.18\text{kN-m}$$

Design procedure

Data required:

1. Moment or loading with span & type of support
2. Width of beam
3. Grade of concrete & steel
4. Spacing of beams.

Step – 1 Preliminary design

From the details of spacing of beam & thickness of slabs the flange width can be calculated from IS code recommendation.

$$b_f = b_w + 6D_f \quad \text{P-37 : IS 456}$$

Approximate effective depth required is computed based on  $l/d$  ratio  $d \approx \frac{l}{20}$  —

Assuming suitable effective cover, the overall depth,  $h = d + C_e$  round off „ $h$ “ to nearest 50mm integer no. the actual effective depth is recalculated,  $d_{\text{provided}} = h - C_e$

Approximate area of steel i.e computed by taking the lever arm as  $Z = d -$

$$M_u = 0.87f_y A_{st} X Z$$

$$A_{st} = \frac{M_u}{0.87f_y X Z}$$

Using this  $A_{st}$ , no. of bars for assumed dia is computed.

Round off to nearest integer no. & find actual  $A_{st}$ .

NOTE: If the data given is in the form of a plan showing the position of the beam & loading on the slab is given as „ $q$ “  $\text{kN/m}^2$  as shown in the fig.

$$W = q \times S \times 1 \text{ also, } b_f \square s$$

Step – 2 N – A depth

The N-A depth is found by trial procedure to start with assume the N-A to be in the flange. Find N-A by equating  $C_u$  &  $T_u$  if  $x_u < D_f$  then NA lies in flange else it lies in web.

In case of NA in the web then find  $\frac{x_u}{d} > 0.2$ , use the equations

for  $C_u$  &  $T_u$  as in case (II) otherwise use case (III).

Compute  $x_{u\text{lim}}$  & compare with  $x_u$ . If  $x_u > x_{u\text{lim}}$  increase the depth of the beam & repeat the procedure for finding  $x_u$ .

Step . 3 Moment of resistance

Based on the position of NA use the equations given in cases (I) or case (II) or case (III) of analysis. For safe design  $M_{ur} > M_u$  else redesign.

Step . 4 Detailing.

Draw the longitudinal elevation & c/s of the beam showing the details of reinforcement.

1. Design a simply supported T – beam for the following data. (I) Factored BM = 900kN-m (II) width of web = 350mm (III) thickness of slab = 100mm (IV) spacing of beams = 4m (V) effective span = 12m (VI) effective cover = 90mm, M20 concrete & Fe 415 steel.

Step . 1 Preliminary design.

$$b_f = b_w + 6D_f \quad l_o = l_e = 12000\text{mm}$$

$$= 350 + 6 \times 100 = 2950 < S = 4000$$

$$h \approx \text{to } \text{---} \text{ (1000 to 800mm)}$$

Assume  $h = 900\text{mm}$

$$d_{\text{provided}} = 900 - 90 = 810\text{mm}$$

$$\text{Approximate } A_{st} = \text{---} = \text{---} = 3279\text{mm}^2$$

Assume 25mm dia bar.

$$\text{No. of bars} = \approx 6.7$$

Provide 8 bars of 25mm dia

$$(A_{st})_{\text{provided}} = 8 \times 491 = 3928\text{mm}^2$$

$$x_{u\text{max}} = 0.48 \times 810 = 388.8$$

Step. 2 N-A depth, Assume NA to be in flange.

$$C_u = 0.36f_{ck} x_u b_f ; T_u = 0.87f_y A_{st}.$$

$$0.36 \times 20 \times x_u \times 2950 = 0.87 \times 415 \times 3927.$$

$$x_u = 66.75 < D_f < x_{u\max}$$

hence assumed position of N-A is correct.

$$\text{Step . 3 . } M_{ur} = 0.36f_{ck}b_f x_u (d - 0.42x_u)$$

$$= 0.36 \times 20 \times 2950 \times 66.75 (810 - 0.42 \times 66.75)$$

$$M_{ur} = 1108.64 \text{ kN-m} > 9.01 \text{ kN-m } M_u$$

Hence ok

Step . 4 Detailing

2. Design a T-beam for the following data. Span of the beam = 6m (effective) & simply supported spacing of beam -3m c/c, thickness of slab = 120mm, loading on slab -5kN/m<sup>2</sup> exclusive of self weight of slab effective cover = 50mm, M20 concrete & Fe 415 steel. Assume any other data required.

3. A hall of size 9mX14m has beams parallel to 9m dimension spaced such that 4 panels of slab are constructed. Assume thickness of slab as 150mm & width of the beam as 300mm. Wall thickness = 230mm, the loading on the slab (I) dead load excluding slab weight 2kN/m<sup>2</sup> (2) live load 3kN/m<sup>2</sup>. Adopt M20 concrete & Fe 415 steel. Design intermediate beam by 1<sup>st</sup> principle. Assume any missing data.

\* 1 inch = 25.4 or 25mm

\* „h“ should be in terms of multiples of inches.

Step . 1 Preliminary design

$$S = 3.55\text{m}$$

$$\text{Effective span, } l_e = 9 + \quad = 9.23\text{m}$$

$$\text{Flange width, } b_f = b_w + 6D_f = 9.23/6 + 0.300 + 6 \times 0.15 = 2.74\text{m} < S = 3.55$$

$$h \approx \text{to} \approx \quad \text{to} \quad = 769.17 \text{ to } 615.33$$

Let us assume  $h=700\text{mm}$ ,  $C_e=50\text{mm}$ .

$$d_{\text{provided}} = 700 - 50 = 650\text{mm}$$

Loading

1. on slab

a) self weight of slab =  $1\text{m} \times 1\text{m} \times 0.15\text{m} \times 25 = 3.75\text{kN/m}^2$

b) Other dead loads (permanent) =  $2\text{kN/m}^2$

c) Live load (varying) =  $3\text{kN/m}^2$   
 (It is known as imposed load)  $q = 8.75 \text{ kN/m}^2$

2. Load on beam

a> From slab =  $9 \times 3.55 = 31.95\text{kN/m}$

b> Self weight of beam =  $0.3 \times 0.55 \times 1 \times 25 = 4.125\text{kN/m}$

$\downarrow$   $\Delta$ depth of web  
 width of beam

$$w = 36.075 \text{ kN/m}$$

$$M = \frac{w l^2}{8} = 383.37\text{kN-m}$$

$$M_u = 1.5 \times 383.4 = 575\text{kN-m.}$$

$$(A_{st})_{app.} = \frac{M_u}{\sigma_{st} \times z}$$

$$= \frac{575 \times 10^6}{235 \times 1000 \times 100}$$

$$= 2769\text{mm}^2 \quad (A_{st})_{actual} = 491 \times 6 = 2946$$



No. of # 25 bars =  $\frac{1000}{3.28} = 5.65 \approx 6$

1 ft of span  $\rightarrow$  depth is 1 inch  $1\text{m} =$

Provide 6 bars of 25mm dia in 2 rows.

Step .2 N-A depth

$$x_{u\max} = 0.48 \times 650 = 312\text{mm}$$

Assuming N-A to lie in flange,

$$x_u = \frac{M_{ur}}{f_{ck} b_f x_u (d - 0.42 x_u)} = 53.91\text{mm} < D_f < x_{u\max}$$

$$M_{ur} = 0.36 f_{ck} b_f x_u (d - 0.42 x_u)$$

$$= 0.36 \times 20 \times 2740 \times 53.91 (650 - 0.42 \times 53.91) / 10^6$$

$$= 667.23\text{kN-m.}$$

Design a T-beam for a simply supported span of 10m subjected to following loading as uniformly distributed load of 45KN/m excluding self wt of the beam by a point load at mid-span of intensity 50KN due to a transverse beam. Assume the width of the beam=300mm & spacing of the beam=3m. Adopt M-20 concrete & Fe 415 steel.

Sol:  $M = \frac{wL^2}{8} + PL$

$$\text{Self cut} = 1 \times 1 \times 0.3 \times 25 = 7.5\text{dKW/M.}$$

$$W = 7.5 + 45 = 52.5\text{KN/M.}$$

Shear, Bond & tension in RCC Beams

Shear

- Types of cracks @ mid span → flexural crack beoz Bm is zero, SF is max
- Type of crack away from mid span → shear f flexural crack.
- Principal tensile stress at supports = shear stress

———— A= area above the point consideration

If  $(A_s)_{hanger} < (A_{st})_{min}$  does not contribute to compression as in doubly reinforced beams.

$(A_{st})_{min} =$  ———

RCC – Heterogonous material → Distribution of shear stress in complex

□ Normal shear stress ———

$$V_u = V_{cb} + V_{ay} + V_d + V_s$$

$$V_u = V_{cu} + V_s.$$

- Shear reinforcement → Vertical stirrup & Bent - up bars

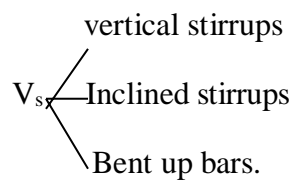
Truss analogy

$$V_{cu} = \tau_{c} \times b \times d$$

$\tau_{c}$  = Design shear stress

IS-456 P-73

$$\tau_{c} \leq \tau_{cmax}$$



$$(A_{sv})_{min} \geq \frac{V_{sv}}{0.87 f_y \sin \alpha}$$

2)  $V_s = V_{us} =$

3)  $V_{us} = 0.87 f_y A_{sv} \sin \alpha$

No(6)

Whenever bent up bars are provided its strength should be taken as less than or equal to  $0.5V_{us}$  (shear strength of reinforcement).

## Procedure for design of shear in RCC

Step; 1 From the given data calculate the shear force acting on the critical section where critical section is considered as a section at a distance „d“ from the face of the support. However in practice the critical section is taken at the support itself.

Step – 2 For the given longitudinal reinforcement calculate  $p_t = \frac{A_{st}}{bd}$ , for this calculate  $T_c$  &  $T_v$  calculate from Pg. 73  $v =$  calculated in step 1.

If  $v > v_{cmax}$  (page 73) then increase „d“

$$V_{cu} = v_c bd$$

If  $V_{cu} \leq V_u$ , Provide min. vertical stirrup as in page 48, clause 26.5.1.6 ie  $(S_v)_{max} \leq$

Else calculate  $V_{us} = V_u - V_{cu}$

Step 3 Assume diameter of stirrup & the no. of leg to be provided & accordingly calculate  $A_{sv}$  then calculate the spacing as given in P-73 clause 40.4(IS-456) This should satisfy codal requirement for  $(S_v)_{max}$ . If shear force is very large then bent-up bars are used such that its strength is less than or equal to calculated  $V_{us}$ .

1. Examine the following rectangular beam section for their shear strength & design shear reinforcement according to IS456-2000.  $B=250\text{mm}$ ,  $s=500\text{mm}$ ,  $P_t=1.25$ ,  $V_u=200\text{kN}$ , M20 concrete & Fe 415 steel

Step 1: Check for shear stress

Nominal shear stress,  $v =$

From table 19, p=73,  $v_c = 0.67\text{N/mm}^2$ .

From table 20, p=73,  $v_{cmax} = 2.8\text{N/mm}^2$ .

$$v_c < v < v_{cmax}$$

The depth is satisfactory & shear reinforcement is required.

Step 2. Shear reinforcement

$$V_{cu} = v_c bd =$$

$$V_{us} = V_u - V_{cu} = 200 - 83.75 = 116.25\text{KN}$$

Assume 2 leg -10mm dia stirrups,  $A_{sv} = 2 \times 100 = 200 \text{ mm}^2$ .

Spacing of vertical stirrups, obtained from IS456-2000

$$S_v = \frac{V_u}{A_{sv} \sigma_{sv}}$$

Check for maximum spacing

i)  $S_{vmax} = \frac{d}{4}$

ii)  $0.75d = 0.75 \times 500 = 375 \text{ mm}$ .

iii) 300mm

$$S_{vmax} = 300 \text{ mm (Least value)}$$

2. Repeat the previous problem for the following data

1)  $b=100 \text{ mm}$ ,  $d=150 \text{ mm}$ ,  $P_t=1\%$ ,  $V_u=9 \text{ kN}$ , M20 concrete & Fe 415 steel

2)  $b=150 \text{ mm}$ ,  $d=400 \text{ mm}$ ,  $P_t=0.75\%$ ,  $V_u = 150 \text{ KN}$ , M25 concrete & Fe 915 steel

3)  $b=200 \text{ mm}$ ,  $d=300 \text{ mm}$ ,  $P_t=0.8\%$ ,  $V_u=180 \text{ kN}$ , M20 concrete & Fe 415 steel.

3. Design the shear reinforcement for a T-beam with following data: flange width = 2000mm. Thickness of flange = 150mm, overall depth = 750mm, effective cover = 50mm, longitudinal steel = 4 bars of 25mm dia, web width = 300mm simply supported span=6m, loading =50kN/m, UDL throughout span. Adopt M20 concrete & Fe 415 steel

Step; [Flange does not contribute to shear it is only for BM]

Step -1 Shear stress

$$V = 150 \text{ KN}$$

$$V_u = 1.5 \times 150 = 225 \text{ KN}$$

$$v = \frac{V_u}{b d}$$

$$A_{st} = 4 \times 491 = 1964 \text{ mm}^2.$$

$$P_g = 73 \quad 0.75 \times 0.56$$

$$P_t = \dots\dots\dots 1.00 \times 0.62$$

- $c = 0.56+$

$= 0.6N/mm^2.$

From table 20, max = 2.8

$\square c < v < c_{max} = 2.8$

Design of shear reinforcement is required

Step – 2 Design of shear reinforcement

$V_{cu} = c b_w d =$

$V_{us} = V_u - V_{cu} = 225 - 126 = 99KN$

Assume 2-L, 8 dia stirrups  $A_{su} = 2 \times 8^2 = 100mm^2$

Spacing of vertical stirrups,

$S_v = \frac{V_{us}}{A_{su} \times \dots}$

Check for Max spacing

i)  $S_{vmax} = \dots$

ii)  $0.75d = 0.75 \times 700 = 525mm.$

iii) 300mm

$S_v < S_{vmax}$   $\square$  provide 2l -#8mm @ 250c/c

Step – 3 curti cement

From similar triangle

$\frac{\dots}{\dots} = \frac{\dots}{\dots}$

- provide (i) 2L – 3 8@ 250 c/c for a distance of 1.4m
- (ii) 2L-#8@300 c/c for middle 3.2m length

**Step.4 Detailing**

Use 2- #12mm bars as hanger bars to support stirrups as shown in fig

A reinforced concrete beam of rectangular action has a width of 250mm & effective depth of 500mm. The beam is reinforced with 4-#25 on tension side. Two of the tension bars are bent up at 45° near the support section in addition the beam is provided with 2 legged stirrups of 8mm dia at 150mm c/c near the supports. If  $f_{ck} = 25\text{Mpa}$  &  $f_y = 415\text{Mpa}$ . Estimate the ultimate shear strength of the support s/n

$$(A_{st})_{xx} = \quad = 982\text{mm}^2.$$

$$P_t =$$

$$P_t = 0.75 \square 0.57$$

$$P_t = 1.00 \square 0.64$$

$$\text{For } P_t = 0.78 \square \quad c = 0.57 +$$

$$c = 0.5784.$$

**1) Shear strength of concrete**

$$V_{cu} = c_{cu}bd = \text{—————}$$

**2) Shear strength of vertical stirrups**

$$(A_{sv})_{stirrup} = 2 \square$$

$$(V_{su})_{st} = \frac{V_u - V_{cu}}{\sin \alpha}$$

$$= \frac{72.3 - 72.3}{\sin 45^\circ}$$

3) Shear strength of bent up bars

$$(A_{su})_{bent} = 2 \times \frac{A_{st}}{\sin \alpha}$$

$$(V_{us})_{bent} = 0.87 f_y (A_{su})_{bent} \sin \alpha$$

$$= \frac{0.87 \times 415 \times 2 \times 314}{\sin 45^\circ}$$

$$= 250.7 \text{ kN}$$

$$V_u = V_{cu} + (V_{us})_{st} + (V_{us})_{bent}$$

$$= 72.3 + 120.35 + 250.7$$

$$V_u = 443.35 \text{ kN}$$

5. A reinforced concrete beam of rectangular s/n 350mm wide is reinforced with 4 bars of 20mm dia at an effective depth of 550mm, 2 bars are bent up near the support s/n. The beam has to carry a factored shear force of 400kN. Design suitable shear reinforcement at the support s/n using M20 grade concrete & Fe 415 steel.

$$V_u = 400 \text{ kN}, b = 350 \text{ mm}, d = 550 \text{ mm}, f_{ck} = 20 \text{ MPa}$$

$$f_y = 415 \text{ MPa}, (A_{st})_{xx} = 2 \times 314 = 628 \text{ mm}^2$$

Step – 1 Shear strength of concrete

$$P_t =$$

$$v_c = \frac{0.52 f_{ck}}{\sqrt{1 + P_t}}$$

$$= 2.07 \text{ MPa}$$

$$v_c < v < v_{c, \text{Max}}$$

Design of shear reinforcement is required.



Step -2 Shear strength of concrete

$$V_{cu} = \phi_c b d =$$

$$V_{us} = V_u - V_{cu} = 323 \text{KN}$$

Step -3 Shear strength of bent up bar.

$$(A_{sv})_{bent} = 2 \times 314 = 628 \text{mm}^2.$$

$$(V_{us})_{bent} = 0.87 f_y (A_{sv})_{bent} \sin \alpha.$$

$$= \frac{0.87 \times 415 \times 628 \times \sin 45^\circ}{1000}$$

$$= 160.3 \text{KN} * +$$

NOTE: If  $(V_{us})_{bent} > V_{us}$  then  $(V_{us})_{bent} = V_{us}$

Step – 4 Design of vertical stirrups

$$(V_{us})_{st} = V_{us} - (V_{us})_{bent} = 323 - 160.3 = 162.7 \text{KN}.$$

Assuming 2L-#8 stirrups

$$(A_{sv})_{st} = 2 \times 50 = 100 \text{mm}^2$$

$$S_v = \frac{V_{us}}{(A_{sv})_{st} \phi_c \sin \alpha} = \frac{162.7 \times 1000}{100 \times 0.87 \times 415 \times \sin 45^\circ} = 540 \text{mm}$$

Provide 2L-#8@120 c/c

$$S_{vmax} =$$

$$0.75d = 412.5 \text{mm}, 300 \text{mm}.$$

$$S_{vmax} = 258 \text{mm}$$

Shear strength of solid slab

Generally slab do not require stirrups except in bridges. The design shear stress in slab given in table 19 should be taken as „k<sub>c</sub>“ where „k“ is a constant given in clause 90.2.1.1

→ The shear stress  $\tau_c < k_c$  hence stirrups are not provided.

→ Shear stress is not required Broz thickness of slab is very less.

Self study: Design of beams of varying depth Page: 72, clause 40.1.1

Use of SP-16 for shear design

SP – 16 provides the shear strength of concrete in table 61 Pg 178 table 62 (179) provides  $(V_{us})_{st}$  for different spacing of 2 legged stirrups of dia 6,8,10 & 12mm. Here it gives the value of in kN/cm where „d“ is in cm. Table 63, Pg 179 provides shear strength of 1 bent up bar of different dia.

Procedure

Step- 1; Calculate  $c = \frac{V_u}{bd}$  & obtain  $c$  from table 61 & also obtain  $c_{max}$  from table 20, Pg 73 –IS956 If  $c < c < c_{max}$  then design of shear reinforcement is necessary. \_

Step - 2  $V_{cu} = c_{st}bd$

$$V_{us} = V_u - V_{cu}$$

Assuming suitable stirrup determine the distance for in kN/cm.

H.W Design all the problems using SP-16 solved earlier.

### Bond & Anchorages

$$F_b = (\sigma_s) (l_d) (\sigma_{bd})$$

$\downarrow$   
 Permitter length

$\swarrow$   
 stress

$$T =$$

$$F_b = T$$

$$l_d \sigma_{bd} =$$

$$l_d = \frac{T}{\sigma_{bd}}$$

$\sigma_{bd}$  = Anchorage bond stress

$\sigma_{bf}$  = flexural bond stress

For CTDs HYSD bars, flexural bond stress is ignored becoz of undulations on surface of steel.

$$\sigma_{bf} = \frac{T}{\Sigma}$$

Z = lever arm

Code requirement

—

Where  $l_d = \frac{T}{\sigma_s}$ ;  $\sigma_s$  = tensile stress in steel

$\sigma_{bd}$  = design bond stress.

The value of this stress for different grades of steel is given in clause 26.2.1.1 →Pg- 93 of code for mild steel bar. These values are to be multiplied by 1.6 for deformed bars. In case of bar under compression the above value should be increased by 25%  $\sigma_s = 0.87f_y$  for limit state design. If  $l_d$  is insufficient to satisfy (1), then hooks or bents are provided. In MS bars Hooks are essential for anchorage

$$\text{Min} = 4\phi$$

$$K = 2 \text{ for MS bars}$$

$$= 4 \text{ for CTD bars}$$

Hook for Ms bars

$$(K+1)\phi$$

Standard 90° bond

Pg – 183 fully stressed = 0.87fig

1. Check the adequacy of develop. Length for the simply supported length with the following data.

(IV) c/s = 25 x 50cm (ii) span=5m (iii) factored load excluding self wt =160KN/m. iv) Concrete M20 grade, steel Fe 415 grade. (v) Steel provided on tension zone. 8 bars of 20mm dia.

Solve:  $C_e = 50\text{mm}$ ,  $h=500\text{mm}$ ,  $d=450\text{mm}$

$$q_{\text{self}} = 0.25 \times 0.5 \times 1 \times 25$$

$$= 3.125\text{KN/m}$$

$$q_{\text{useful}} = 1.5 \times 3.125 = 4.6875\text{KN/m.}$$

$$\text{Total load} = 60 + 4.6875$$

$$= 64.6875\text{KN/m}$$

$$V_u = \text{————} = 161.72\text{KN}$$

$$X_{u\text{lim}} = 0.48 d = 0.48 \times 450 = 216\text{mm}$$

$$M_{u\text{lim}} = 0.36f_{ck} x_u b (d - 0.42x_u) 10^6$$

$$= 139.69\text{kN-m}$$

Let  $W_s = 300$ ,  $l_o = 150\text{mm}$ .

$$l_0 + l_e = +0.15 = 1.01m$$

$$L_d = \dots$$

$$b_d = 1.6 \times 1.2 = 1.92 \quad \text{Table R43.}$$

$$l_d = \dots$$

--

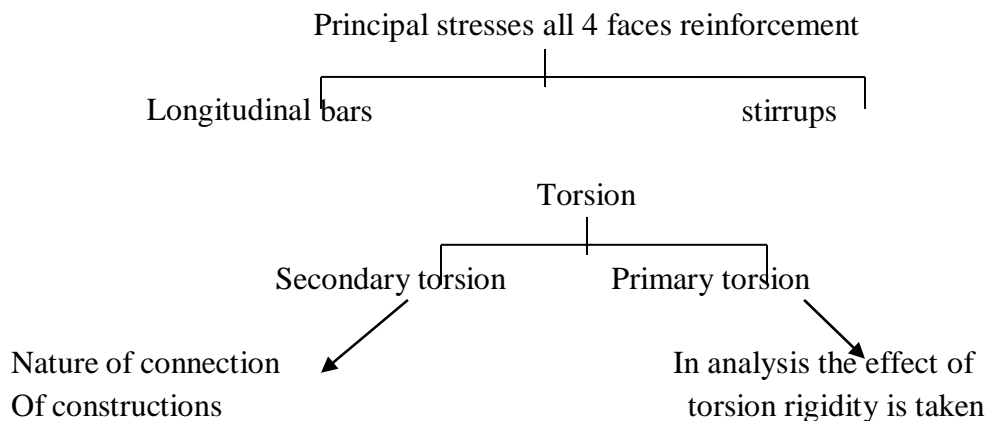
2. A cantilever beam having a width of 200mm & effective depth 300mm, supports a VDL hug total intensity 80KN(factored) 4nos of 16mm dia bars are provided on tension side, check the adequacy of development length ( $l_d$ ), M20 & Fe 415.

Design for torsion

-- -- --

$$t_{max} = \dots$$

For the material like steel



to account

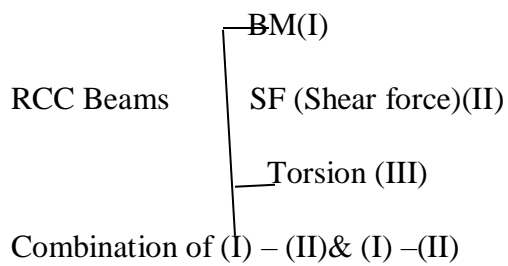
Eg. Chejja or sunshade L-window

Cross – cantilever  
Eg- for Secondary torsion

Eg. (1) Plan of Framed Structure  
Primary torsion

(3) Ring beam in elevated tank

(2) Arc of a circle



IS-456 →Pg – 79 Procedure

(1) Flexure & Torsion

$$M_{e1} = M_u + M_T$$

$$M_T = T_u ($$

D= Overall depth, b=breadth of beam  
 Provide reinforcement for Mt in tension side  
 If  $M_T > M_u$  provide compression reinforcement for

(2) Shear & Torsion

$$V_e = V_u + \text{---}$$

$$v = \text{---}$$

Shear reinforcement

$$A_{sv} = \text{---}$$

$$= \text{---}$$

Pg.48 → clause 26.5.1.7

$S_{V_{max}}$  is least of

i)  $x_1$

ii)  $\text{---}$

iii) 300mm

$$A_{sw} > \text{---}$$

1. Design a rectangular reinforced concrete beam to carry a factored BM of 200KN-m, factored shear force of 120kN & factored torsion moment of 75 KN-m Assume M-20 concrete & Fe 415 steel

Sol:  $M_u = 200\text{KN-m}$ ,  $T_u = 75\text{KN-m}$ ,  $V_u = 120\text{K.N}$ .

Step -1= Design of BM & Torsion

Assume the ratio =2

$$M_T = \text{---}$$

No compression reinforcement design is necessary

$$M_T < M_u$$

$$M_{e1} = M_T + M_u = 200 + 132.35 = 332.35 \text{ KN-m.}$$

$$d_{bal} = \sqrt{\frac{M_{e1}}{0.138 f_{ck} b}} = 633.5 \text{ mm.}$$

Assume  $b=300\text{mm}$ ,  $\rho_{lim}=2.76$

Assuming overall depth as  $700\text{mm}$  & width as  $350\text{mm}$  & effective cover  $=50\text{mm}$ .

D Provided  $=700-50$

$$=650\text{mm}$$

Area of steel required for under reinforced section,

$$P_t = \left[ \frac{M_{e1}}{b d^2} \right]$$

$$= \left[ \frac{332.35 \times 10^6}{350 \times 650^2} \right]$$

$$= 0.73 < 0.96$$

( $p_t \text{ lim}$ )

$$\rho A_{st} =$$

$$\text{Assume } 25\text{mm dia bars } = 3.38 \approx 4$$

Provide 4 bars -#25 dia

$$(A_{st}) \text{ provided } = 1963 \text{ mm}^2$$



Step – 2 Design for shear force & torsion

$$V_e = V_u + \dots$$

$$V_e =$$

$$v_e = 2.03 < 2.8 (T_{cmax}) \text{ P-73}$$

$$P_t =$$

Table – 19  $c = 0.56 +$

Assuming 2 – legged = 12mm dia;

$$A_{su} = 2 \times 12^2 = 226\text{mm}^2$$

From IS-456;Pg-75

$$S_v =$$

$$b_1 = 275\text{mm}, d_1 = 600\text{mm}$$

$$y_1 = 600 + 25 + 2 \times 6 = 637\text{mm}$$

$$x_1 = 275 + 25 + 2 \times 6 = 312\text{mm}.$$

$x_1, y_1$  □ dist of centre of stirrups.

Provide 2 - #12@ top as hanger bars

$$S_v =$$

Check

$$1. A_{sv} > \dots$$

$$226 > 210.84$$

$$2. S_{vmax} \text{ a) } x_1 = 312\text{mm} \quad \text{b)-} \quad \text{c) } 300\text{mm}$$

$$\text{d) } 0.75d = 487.5$$

As  $D=h>450$  , provide side face reinforcement

$$A_{sw} = x \ 350 \times 650 = 227\text{mm}^2$$

Provide 2-#16 bars ( $A_{st} = 400\text{mm}^2$ ) as side face

2. Repeat the same problem with  $T_u=150\text{KN-m}$  & other data remain same

$$\text{Solve } M_u = 200\text{KN-m}, T_u=150\text{KN-m}, V_u=120\text{KN}$$

Step – 1 Design of EM & torsion

Assume the ratio  $D/b = 2$ .

$$M_T = \frac{150}{1.7} = 264.70\text{KN-m}$$

$M_T > M_u$  Compression reinforcement is required.

$$M_G = M_T + M_u = 264.7 + 200 = 464.7\text{KN-m}$$

$$d_{bal} = \sqrt{\frac{M_G}{f_{ck} b}}$$

Assume  $b = 300\text{mm}$ ,  $\rho_{lim} = 2.76$ .

Assuming overall depth as  $800\text{mm}$  & width as  $400\text{mm}$  & effective cover =  $50\text{mm}$ .

$$D_{provided} = 800 - 50 = 750\text{mm}$$

Area of steel required for under reinforced section,

$$P_t = \left[ \frac{M_G}{f_{ck} b D^2} \right]$$

$$= \left[ \frac{464.7}{10 \times 300 \times 750^2} \right]$$

$$P_t = 0.66 < 0.96(p_{tlim})$$

Assume 25mm dia bars = = 5.86≈6.

Provide 6 bars - #25 dia

$$A_{st2} = 255.99$$

$$A_{st} = 3135.99 \square 7 - \#25.$$

$$M_{e2} = 264.7 - 200 = 64.7 \text{KN-m.}$$

$$A_{sc} = \text{---}$$

$$A_{st1} = A_{st2} = \text{---}$$

$$A_{st} = kb \text{ of bars}$$

Step – 2 Design for shear force torsion

$$V_e = v_u + \text{---}$$

$$= 120 +$$

$$v_e =$$

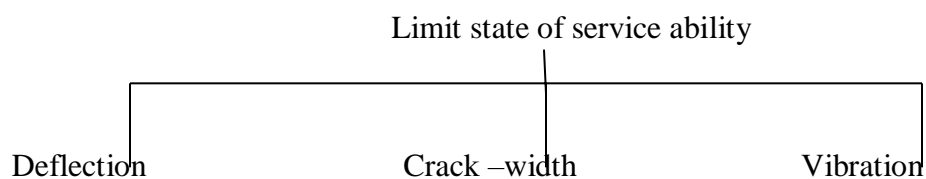
$$P_t = 0.98 \quad c = 0.61$$

Assuming 2 legged 12mm dia

$$A_{sv} = 2 \text{ ---}$$

From IS-456, P<sub>g</sub> – 75

$$S_v = \text{---}$$



1. Deflection  $\rightarrow$  Span to effective depth ratio  
 $\rightarrow$  Calculation

$$y_{\max} = \frac{w l^4}{8 E I}$$

$$y_{\max} \leq \frac{l}{200}$$

As control of deflection by codal provision for l/d ratio

Cause 23 .2.1 Pg – 37 of IS 456 – 2000

Type of beam	l/d ratio	
	Span, $l \leq 10\text{m}$	Span $> 10\text{m}$
i) Cantilever beam	7	Should be calculated
ii) Simply supported beam	20	_____
iii) Continuous beam	26	_____

Effect on l/d ratio

1. Tension reinforcement :  $> 1\%$

$$p_g - 38; f_s = 0.58 f_y$$

SP-24 → Explanatory hand book

$$M_{ft} = [0.225 + 0.003225 f_s + 0.625 \log_{10}(p_t)]^{-1} \leq 2$$

2. Compression reinforcement.

$$M_{fc} = \frac{M_u}{\phi} + \frac{M_{cr}}{\phi}$$

3. Flange action or effect

$$M_{fl} = 0.8 M_u$$

$$= 0.8 M_u + \frac{M_{cr}}{\phi}$$

$$- \quad (-)$$

Design

1. Flexure + torsion
2. Check for shear + Torsion, bond & Anchorage
3. Check for deflection

1. A simply supported R-C beam of effective span 6.5m has the C/S as 250mm wide by 400mm effective depth. The beam is reinforced with 4 bars of 20mm dia at the tension side & 2-bars of 16mm dia on compression face. Check the adequacy of the beam with respect to limit state deflection, if M20 grade concrete & mild steel bars have been used.

$$B=250\text{mm}, d=400\text{mm}, f_{ck}=20\text{Mpa}, f_y=250\text{Mpa}$$

$$A_{st} = 4 \quad = 1256 \text{ m}$$

$$A_{sc} = 2 \quad = 402 \text{ m}$$

$$P_t = \text{—————}$$

$$P_l = \text{—————}$$

From Pg – 37,  $(l/d)_{\text{basic}} = 20$

$$f_s = 0.58 \times 250 \times 1 = 145$$

$$m_{ft} = [0.225 + 0.003225 \times 145 + 0.625 \log_{10}(1.256)]^{-1} = 1.325$$

$$m_{fc} = * +$$

$$m_{fl} = 1 \text{ (rectangular section)}$$

( Required =

Check

$$(- \quad \text{—————})$$

- 2) Check the adequacy of a T-beam with following details (i) Web width (wb) = 300mm, (ii) Effective depth (d) = 700mm (iii) flange width (bf) = 2200mm (iv) effective span of simply supported beam(l) =8m (v) reinforcement a) tension reinforcement – 6bars of 25 dia b) compression reinforcement – 3 bars of 20 dia (vi) Material M25 concrete & Fe 500 steel.

Deflection calculation-Short term deflection

Long term deflection (shrinkage, creep)



## 1. Short term deflection

$$E_c = 5000 \sqrt{f_c}$$

Slope of tangent drawn @ origin → Tangent modulus

Slope of tangent drawn @ Specified point → secant Modulus 50% of Material

$I_{gr}$  — For elastic;  $I_{ef}$  → cracked section

Pg. 88  $I_{eff} =$  ( )

$I_r$  = Moment of inertia of cracked section

$M_r$  = cracking Moment =

- NA → stress is zero
- CG → It is point where the wt. of body is concentrated
- $Y_t \neq x$
- $M$  = Max. BM under service load:  $Z$  = lever arm

$X$  = depth of NA :  $b_w$  = width of web:  $b$  = width of compression face

(For flanged section  $b = b_f$ )

For continuous beam, a modification factor  $\alpha_e$  given in the code should be used for  $I_r$ ,  $I_{gr}$ , &  $M_r$ . The depth of NA „ $x$ “ & lever arm  $Z$  has to be calculated by elastic analysis is working stress method explained briefly below.

Introduction to WSM

$M = \frac{P l^2}{8}$  ————— Permissible stress  $P$  g.80

From property of similar triangles,

$$\frac{f_c}{x} = \frac{P}{d}$$

$$f_c = E_c \times \epsilon_c \quad \text{--- 2}$$

$$f_s = E_s \epsilon_s = m E_c \epsilon_c \quad \text{--- 3a.}$$

$$C = T.$$

$$A_{sc} = \frac{m A_{st} x}{d - x}$$

$$\frac{1}{2} [ [$$

Eq (4) can also be written in the form of

$$\frac{m A_{st} x}{d - x} = \frac{P l^2}{8 E_c x}$$

In eq.5, the modular ratio for compression steel is taken as (1.5m)

Use of SP-16 for calculating  $I_{ef}$

1. Using Table – 91: Pg – 225-228, we can find NA depth for simply reinforced,  $P_c = 0$
2. Using Table – 87-90, find out cracked moment of inertia  $I_r$
3.  $I_{eff}$  chart -89, Pg.216

Cracked moment of inertia can be found by the following equation.

For singly reinforced section

$$I_r = \frac{b x^3}{3} + m A_{st} x (d - x)$$

For doubly reinforced section,

$$I_r = \frac{b x^3}{3} + m A_{st} x (d - x)^2 + m A_{sc} (d - x)^2$$

————— Shrinkage deflection



## 2. Long Term deflection

— Creep effect deflection

a) shrinkage deflection

reduces stiffness (EI)

$\epsilon_{sh} = 0.004$  to  $0.0007$  for plain concrete

$= 0.0002$  to  $0.0003$  for RCC

$$Y_{sh} = k_2 \epsilon_{cs} l^2$$

$K_3 = \text{cantilever} - 0.5$

Simply supported member –  $0.125$ .

Continuous at one end –  $0.086$  Pg-88

Full Continued –  $0.063$

□

√

—————  
√

b) creep deflection → permanent

$y_{scp} = \text{Initial deflection} + \text{creep deflection using } E_{cc} \text{ in place of } E_c \text{ due to permanent } E_{se}$

$C_c = \text{Creep co-efficient}$

$1.2$  for 7 days loading

$1.6$  for 28 days loading

$1.1$  for 1 year loading

$Y_{sp} = \text{Short per deflection using } E_c$

$$Y_{cp} = Y_{scp} - Y_{sp}$$

A reinforced concrete cantilever beam 4m span has a rectangular section of size 300 X 600mm overall. It is reinforced with 6 bars of 20mm dia on tension side & 2 bars of 22mm dia on comp. side at an effective cover of 37.5mm. Compute the total deflection at the free end when it is subjected to UDL at service load of 25KN/m, 60% of this load is permanent in nature. Adopt M20 concrete & Fe 415 steel.

Sol:

$$A_{st} = 6 \times \quad = 1885m \quad \text{---}$$

$$A_{sc} = 2 \times \quad = 760m$$

$$f_{ck} = 200\text{Mpa}, f_y = 415\text{Mpa}, E_s = 2 \times 10^5\text{Mpa}.$$

$$E_c = 5000\sqrt{\quad}$$

$$f_{cr} = 0.07\sqrt{\quad}$$

$$m = \text{---} \quad \text{---}$$

$$I_g =$$

$$Y_t = \text{---} \quad \text{---}$$

Step : 1 Short term deflection

$$y_{short} = \text{---}$$

$$I_{eff} = \text{---}$$

$$M_{cr} = \text{---} \quad \text{---} \quad \text{---}$$

$$M = \text{---} \quad \text{---}$$

$$\text{---} \quad \text{---}$$

From equilibrium condition

$$\text{---}$$

\_\_\_\_\_

$$x^2 - 52.58x - 64705.5 = 0$$

$$x = 189.28 \text{ mm}$$

$$Z \approx d - \text{_____}$$

$$I_r = \frac{2}{3} mA (d-x)^2 \quad \text{st}$$

$$= 3.1645 \times 10^9 \text{ mm}^4$$

$$I_{\text{eff}} = \frac{\text{_____}}{\text{( )}}$$

$$= 3.01 \times 10^9 \text{ mm}^4$$

$$Y_{\text{short}} = \frac{\text{_____}}{\text{_____}}$$

$$= 11.31 \text{ mm} ; I_{\text{cr}} \square I_{\text{eff}} \square I_g$$

$$\square I_{\text{eff}} = I_{\text{cr}} = 3.1645 \times 10^9 \text{ mm}^4.$$

Step – 2 long term deflection

a) Due to shrinkage

$$Y_{\text{cs}} = k_3 \square_{\text{cs}} l^2$$

$$K_3 = 0.5 \text{ for cantilever } \text{_____}$$

$$P_t - P_c = 1.117 - 0.45 = 0.667 < 1.0$$

$$K_4 = \frac{\text{_____}}{\sqrt{\text{_____}}}$$

$E_{\text{cs}}$  = shrinkage strain = 0.0003 (Assumed value)

$$\square_{\text{cs}} = \frac{\text{_____}}{\text{_____}}$$

$$Y_{cs} = 0.5 \times 2.2 + 10^{-7} \times 4000^2 = 1.82\text{mm.}$$

b) Due to creep

$$E_{cc} = \quad [$$

=

$$Y_{scp} = \text{—————}$$

$$= \text{—————}$$

$$Y_{sp} = 0.6 y_{short} = 6.8\text{mm.}$$

$$Y_{cp} = 17.64 - 6.8 = 10.84$$

$$Y = y_{short} + y_{cs} + y_{cp} = 23.97$$

$$= 11.31 + 1.82 + 10.84$$

Unsafe

Doubly reinforced section

—————

$$M_d =$$

$$M_l = \text{—} \quad \text{—}$$

$$M_u = 1.5(M_D + M_C) = 1.5(17.58 + 100) = 176.37\text{KN-M}$$

$$X_{ulim} = 0.48 \times d = 324\text{mm}$$

$$M_{ulim} = Q_{lim} b d^2$$

$$= \text{—————}$$

$$= 565.88 \text{KN-m.}$$

Singly reinforced s/n  $A_{st} = - \text{—————}$

$$P_{lim} = 7.43\% = \text{—————}$$

6 -#25bar is taken.

$$(A_{st})_{provided} = 6 \times 490 = 2940 > 2895$$

2b)  $b=150\text{mm}$ ,  $d=400\text{mm}$ ,  $p_t=0.75\%$ ,  $v_v=150\text{KN}$ ,  $f_{ck} = 25$ ,  $f_y=415$

$$v = \text{—————}$$

$$c = 0.57 \text{N/mm}^2.$$

$$c_{max} = 3.1 \text{N/mm}^2.$$

$$c < v < c_{mac}$$

$$V_{cu} = c b d =$$

$$V_{us} = V_u - V_{cu} = 115.8 \text{KN.}$$

Assume 2L-10#  $A_{su} = 2 \times -$

$$= 157.07 \text{mm}^2.$$

$$S_v = \text{—————}$$

$$= \text{—————}$$

$$= 195.88 \text{mm}$$

$$S_{vmax} = \text{—————}$$

$$0.75d = 300\text{mm}$$

$$300\text{mm}$$

□ provide 2l-#10@ 195.99 c/c.

2c) b=200mm, d=300mm,  $p_t=0.8\%$ ,  $v_v=180\text{KN}$ ,  $f_{ck}=20$ ,  $f_y=415$ .

$$c = \text{—————}$$

$$c_{max} = 2.8 \quad p_t 0.75 \quad c 0.56$$

$$1.0 \quad 0.62$$

$$P_f = 0.8; \quad = 0.56 + \text{—————}$$

$$c = 0.572 \text{ N/mm}^2.$$

$v > c_{max}$ ; unsafe, hence increased

Let „d“ be 350mm

$$v = 2.57\text{N/mm}^2.$$

□  $c < v < c_{max}$ ; safe sh.rei required.

Step 2 shear reinforcement.

$$V_{cu} = c_{bd} = 0.572 \times 200 \times 350 = 40\text{KN}.$$

$$V_{us} = v_u - V_{cu} = 180 - 40 = 140\text{KN}.$$

Assume 2L-#10;  $A_{sv}=157\text{mm}^2$ .

Spaces of vertical stirrups

$$S_v = \text{—————}$$

Step 3: Check for max. spacing

1.  $S_{vmax} = \text{—————}$
2.  $0.75d = 0.75 \times 350 = 262.5\text{mm}$
3.  $300\text{mm} \quad S_v < S_{umax}$  is  $140 < 262.5\text{mm}$

Hence provide 2L-#10@140 c/c.

4)  $l_e = 10m$

$W = q^1 + q^{11} l = 45 \text{ KN/m}$

$b_w = 300mm, s = 3m, f_{ck} = 20Mpa, f_y = 415Mpa$

$b_f =$

$=$

$= 2566.67mm < 3000mm$

$h \equiv \quad - [$

$h = 750mm, d = 700mm$

b x h x l density

$self = 0.3 \times 0.65 \times 1 \times 25$

$q^{11} = 4.875 \text{ KN/m}$

$W = 45 + 4.875 = 49.875 \text{ KN/m}$

$M = \text{—————} \quad \text{————} \quad \text{————}$

$M_u = 1136.7 \text{ KN-m}$

$(A_{st})_{oppr} =$   
 $( ) -$

~~Step 2:~~  $x_u =$

$M_{ur} = 0.36 f_{ck} x_u b_f (d - 0.42 x_u) = 1169.4 \text{ KN-m}$

2. Step 2 :  $b_f =$  —

$=$  —

$= 2020mm$

$h \equiv \quad - [^2$

$h = 450mm \quad q = 8.5 \text{ KN/m}^2$

$$d = h - 50 = 400 \text{ mm} \quad w - q \times s \times l = 8 \times 3 \times 1$$

$$(A_{st})_{app} = \quad ( \quad \rightarrow )$$

$$= \frac{\quad}{\quad * \quad} + \quad -$$

$$= 824.80 \text{ mm}^2.$$

$$\text{Provide } 3\text{-}\#20 \quad \square (A_{st})_{provided} 3 \times 314 = 942 > 824.8$$

Step 2 : Assume NA to be on flange

$$C_u = 0.36 f_{ck} b_f x_u$$

$$T_u = 0.87 f_y A_{st}$$

$$x_u = \text{—————}$$

$$x_{u\text{lim}} = 192 \text{ mm}.$$

$$M_{ur} = 0.36 f_{ck} b_f x_u (d - 0.42 x_u)$$

$$= 0.36 \times 20 \times 2020 \times 23.38 [400 - 0.42 \times 23.38]$$

$$= 132.67 \text{ KN/m} < 101.25 \text{ KN-m}.$$

SECTION –A

(Very Short Answer Type Question )

- a) Reinforcing material
- b) Characteristic strength
- c) Nominal shear stress
- d) Anchorage
- e) Effective span of a beam
- f) Critical neutral axis
- g) Design value
- h) T beam & L beam
- i) Column
- j) Axial load
- k) Pedestal
- l) Pre-stressed concrete
- m) Name any two system of pre-stressing
- n) Factored load
- o) Singly reinforced beam
- p) One way slab
- q) Moment of resistance
- r) Lever arm



## SECTION- B

(Short Answer Type Questions)

Q- I - What are the properties of HYSD steel ?

Q-II – Why steel is used as reinforcing material ?

Q-III- Why Limit state Design is considered more rational than working stress diagram

Q-IV-What are the various types of shear reinforcement used

Q-V- Why HYSD bar used do not require hook at the end for anchorage

Q-VI Difference between balanced, under reinforced & reinforced section for WSM

Q-VII- State the assumption made in limit state of collapse

Q- VIII- Why over reinforced section are not allowed in limit state method of design

Q- IX- When side face reinforcement is provided in a beam? What is the specification for side face reinforcement

Q-X- Difference between under reinforced and over reinforced beam

Q-XI- How will you check weather a beam of given dimension has to be designed as doubly reinforced beam

Q-XII- Write the function of longitudinal & traverse reinforcement

Q-XIII-Why slab are normally safe in shear

Q-XIV- Why main reinforcement is provided along both the direction in a two way slab

Q-XV- Difference between pre tensioning and post tensioning

## SECTION-C

(Long Answer Type Questions)

Q-1- An rectangular RCC beam of size 250 mm wide and 450 mm effective depth is reinforced with 3 bar of 16 mm dia the beam subjected to a bending moment of 120 KN-m find the stresses developed in steel & concrete  
Take  $m= 13.33$